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Dielectric Properties of a Heterogeneous Plasma*

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(Abstract). In this report, the author develops a mathematical expression for the tensor of dielectric permeability of a heterogeneous plasma being confined by a constant magnetic field. The principal attention in the calculation of this tensor is directed to a consideration of the effects linked with the drift motions of the plasma particles, which (effects) are appreciable for waves with a very low phase velocity, i.e. are of the order of the velocities of the drift movements. In the paper, the concept is expressed that a final understanding of the computational tensor of dielectric permeability is very convenient for a solution of the problems involving the fluctuations and stability of heterogeneous plasma since under such an approach there is required only a solution of the Maxwell equation, without having to resort each time to solutions of the equations for motion of charges. A calculation of the tensor of dielectric permeability is conducted for the case of a "plane geometry" plasma, i.e. it is postulated that the force lines of a magnetic field are straight lines, parallel to one another. For the purpose of a qualitative consideration of the force lines' curvature, a fictitious gravitational field is introduced. It is hypothesized that the gradients of the equilibrium values and also the gravitational field are sufficiently small and are directed transversely to the magnetic field. The frequency of oscillations is assumed to be high in comparison with the frequency of the particles' collisions.

An expression for the tensor of dielectric permeability is found with the aid of the Vlasov equation. The equilibrium state of each type of plasma charge is typified by the function of the centers of Larmor circles of arbitrary type.

The frequency of a wave is assumed to be arbitrary with respect to the cyclotron frequency of any type of particles, while the wave length is regarded as arbitrary in comparison with the Larmor radius of the particles (but as less than the dimension of the plasma's heterogeneity). The sense of the wave vector is assumed to be arbitrary with respect to the directions of the magnetic field and the plasma's heterogeneity. Consideration is given both to the macroscopic (Larmor) drift velocities of the type $c\nabla p/e\,n\,H$, and to the microscopic drifts of individual particles—diamagnetic, gravitational, and electrical. In particular, the consideration of the diamagnetic drift, associated with the gradient of the magnetic field, permits us to examine the problems regarding the fluctuations not only of low pressure ($\beta \equiv 8\pi\, p/H \ll 1$) but also of high pressure plasma ($\beta \approx 1$).

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A detailed examination is made of the case of low-pressure plasma ($\beta \leqslant 1$) in the absence of equilibrium electrical and gravitational fields when a significant role is played only by the effects of Larmor drift. For this case, there is derived a more simple expression for the tensor of dielectric permeability and also an equation for the non-vortical oscillations of arbitrary frequency.

Particular attention is devoted to the low-frequency waves (with a frequency lower than the cyclotron ionic), propagating almost transverse to the magnetic field. Dispersion equations are developed for Alfvén and ion-sonic waves in a heterogeneous plasma, modified by drift movements. These waves prove to be unstable and possibly their oscillation buildup may represent a danger in the experiments on the confinement of the plasma.

1 Introduction

The problem of the oscillations of a heterogeneous has attracted the attention of numerous researchers [refs. 1 - 4]. This is explained by the fact that certain types of oscillation prove unstable and can affect significantly the transfer coefficients in the plasma. However at present the methods of investigating this problem have been developed much less than is the case for homogeneous plasma [refs. 5 - 8]. In this report, a general method is developed for studying the oscillations with the aid of the tensor of dielectric permeability. This tensor is computed below by way of the solution of the kinetic equation. If the tensor of dielectric permeability is known, the problem of investigating an arbitrary type of oscillation reduces only to a solution of the Maxwell equation; therein, the need is obviated of solving each time the equations for the motion of the charges. Such a method also permits us to systematize the individual, at first glance disconnected, types of oscillations of a heterogeneous, and also to establish their interrelationship with the types of oscillation in a homogeneous plasma. According to the form of the various components of the tensor, even prior to a solution of the Maxwell equation, we sometimes are able to form a conclusion regarding the existence and the properties of a given type of oscillations.

In this report, we limit ourselves to a survey of the case of "plane geometry", i.e. we consider the force lines of a permanent magnetic field to be straight parallel lines. For the purpose of a qualitative consideration of the curvature of the force lines, a fictitious gravitational field is introduced. (As is known, the effects being evoked by these two factors are analogous; e.g., convective instability [5]). The gradients of the equilibrium values and also the gravitational field are assumed to be sufficiently small and directed transversely to the magnetic field. The frequency of oscillations is assumed to be high as compared with the frequency of the particles' collision. A solution of the kinetic equation is sought in a linear approximation in respect to the wave amplitude.

In Section 2 below, we have found the distribution function with an accuracy up to terms of the first order in respect to the gradients of the equilibrium parameters of the plasma. The corresponding equilibrium distribution function is considered to be an arbitrary functions of the velocities and of one of the coordinates (but, of course, as depending only slightly on this coordinate). The wave frequency is postulated to be arbitrary as compared with the

cyclotron frequency of the type of particles being considered. The dependence of a wave's field upon the coordinates is selected in the form of the product resulting from multiplying the plane wave $e^{ik \cdot r}$ by an arbitrary function of the direction of the heterogenity ("amplitude"). Such a choice permits the finding of the local characteristics of the plasma, as of a dielectric medium. In the expression for the perturbed function of distribution, there are contained the first derivatives with respect to the coordinate of the wave amplitude. The ratio of the Larmor radius of the particles to the "wave length" (i.e. to the value $\sim 1/|k|$) is assumed to be arbitrary. The wave length is considered to be slight as compared with the dimension of the system.

In Section 3, the expression developed for the distribution function is used for the calculation of the conductivity tensor of a heterogeneous plasma, $\sigma_{\alpha\beta}$, and also of the polarization vector X, yielding a connection between the density of the charge and the wave's dielectric field. Specifically, the vector X is useful in the investigation of the longitudinal oscillations ($\nabla \times E \approx 0$), when in place of the complete system of the Maxwell equation, we can use the Poisson equation, in which there enters only the charges' density. The vector X is linked with the tensor $\sigma_{\alpha\beta}$ by the continutiy equation and equals a certain combination of components $\sigma_{\alpha\beta}$. However, the study of the longitudinal oscillations can not be correctly conducted solely with the aid of the Poisson equation, since it is required each time to substantiate the hypothesis regarding the longitudinal state, and in this regard one can by no means manage without the complete system of Maxwell equations, in which are included the currents and hence all the components of the tensor $\sigma_{\alpha\beta}$.

The linear approximation in respect to the gradients of the plasma's equilibrium parameters does not always prove adequate for the investigation of certain wave types. Therefore, in Section 4, we have solved the kinetic equation with an accuracy up to terms, quadratic in respect to the gradient of an equilibrium distribution function, for a particular instance of low pressure plasma. It is demonstrated that these terms may become real for waves of very low (as compared with the cyclotron) frequency. Such frequencies are typical of magnetohydrodynamic instabilities [ref. 2].

The results of Sections 2 - 4, having great generality, are at the same time quite awkward. They are represented in the form of infinite sums of double integrals in respect to the velocities. If we choose a certain concrete form of equilibrium distribution function, in the expressions for $\sigma_{\alpha\beta}$ we can then conduct an integration with respect to the velocities. In Section 5, we have chosen a Maxwell equilibrium distribution function (with the appropriate slight addition, associated with the plasma's heterogeneity) and also assuming that the plasma's pressure is low ($8\pi\ p/B_0^2 \ll 1$; $\nabla B_0 = 0$; $g = 0$), we conducted in $\sigma_{\alpha\beta}$ the integration with respect to velocities. In this regard, the tensor $\sigma_{\alpha\beta}$, describing the properties of a heterogeneous plasma is expressed ina a form analogous to the case of a homogeneous plasma—as the infinite sum from the product of the Cramp (Kramp) functions times the Bessel function from the imaginary argument [ref. 2]. In this section, it is shown that if the effects linked with the gradient of the magnetic field and the gravity forces are not substantial, in the reading-off system where the equilibrium electric field equals zero, the tensor of dielectric permeability, $\varepsilon_{\alpha\beta} \equiv \delta_{\alpha\beta} + 4\pi i \sigma_{\alpha\beta}/\omega$, consists of the sum of the three terms, one of which is the tensor $\varepsilon_{\alpha\beta}$ for the homogeneous plasma, the other is linked with the heterogeneity of the wave am-

plitude, and the third is associated with the gradient of the equilibrium distribution function. Taking into account the importance of the case of low-frequency waves, when the heterogeneity effects are particularly appreciable, we wrote in Sect. 5 an expression for $\varepsilon_{\alpha\beta}$ for this case as well. In addition to $\varepsilon_{\alpha\beta}$ for the Maxwellian plasma (low pressure type), an expression is also adduced for the projection of the polarization vector X for the direction of the wave vector, i.e. an expression for the charges' density.

In Sect. 6, we have derived a dispersion equation described the Alfven and ionic-sonic waves in a low pressure heterogeneous plasma.

2 Solution of the Kinetic Equation

We assume that the plasma is located in the constant magnetic and electric fields B^o, E^o, the gravitational field g and in the field of the electromagnetic wave E, B. If we postulate that the motion of each charged plasma particle for a certain time interval is entirely determined by these fields, and not by the collisions with other particles, within this time interval of distribution function of each type of particles, f, is described by the known Vlasov equation:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}+\Big[\frac{e}{m}\mathbf{E}+\frac{e}{mc}\mathbf{v}\times\mathbf{B}+\frac{\mathbf{F}}{m}$$

$$+\mathbf{v}\times\boldsymbol{\omega}_c\Big]\frac{\partial f}{\partial \mathbf{v}}=0 \qquad (2.1)$$

Here $F = e\,E^o + m\,g$, $\omega_c = e\,B^o/mc$; the remaining notations are conventional. Inasmuch as Eq. 2.1 is a linear homogeneous differential equation in partial derivatives, its solution can reduce to the solving for the constants of the motion of the corresponding system of ordinary differential equations:

$$\frac{d\mathbf{v}}{dt}=\frac{e}{m}\mathbf{E}+\frac{e}{mc}\mathbf{v}\times\mathbf{B}+\frac{\mathbf{F}}{m}+\mathbf{v}\times\boldsymbol{\omega}_c$$

$$\frac{d\mathbf{r}}{dt}=\mathbf{v} \qquad (2.2)$$

The latter ones are handy to solve, after having selected a cylindrical system of coordinates in a velocity field with a polar axis along the magnetic field B^o and a Cartesian system in an ordinary space. Then Eq. 2.2 can be written over in the form:

$$\frac{d\varepsilon_\perp}{dt}=e\,\mathbf{E}\cdot\mathbf{v}_\perp+\frac{e\,\mathbf{B}}{c}\cdot\mathbf{v}_\perp\times\mathbf{v}_\parallel+\mathbf{F}\cdot\mathbf{v}_\perp$$

$$\frac{d\varepsilon_\parallel}{dt}=e\,\mathbf{E}\cdot\mathbf{v}_\parallel-\frac{e\,\mathbf{B}}{c}\cdot\mathbf{v}_\perp\times\mathbf{v}_\parallel$$

$$\frac{d\varphi}{dt}=-\omega_c+\frac{1}{mv_\perp}(-F_x\sin\varphi+F_y\cos\varphi)$$

$$+\frac{e}{mv_\perp}\Big[-\Big(E_x-\frac{v_\parallel}{c}B_y\Big)\sin\varphi$$

$$+\Big(E_y+\frac{v_\parallel}{c}B_x\Big)\cos\varphi\Big]-\frac{eB_z}{mc} \qquad (2.2a)$$

(Eq. cont'd)

$$\frac{dx}{dt}=v_\perp\cos\varphi;\quad \frac{dy}{dt}=v_\perp\sin\varphi;\quad \frac{dz}{dt}=v_z;$$

$$v_\perp=(v_x^2+v_y^2)^{1/2}\qquad v_\parallel=v_z,\qquad \varphi=\arctan\frac{v_y}{v_x},\tag{2.2a}$$

$$\varepsilon_\perp=\frac{mv_\perp^2}{2},\qquad \varepsilon_\parallel=\frac{mv_\parallel^2}{2}$$

It is assumed that the orientation of heterogeneity of the plasma's equilibrium parameters lies in the plane x, y and forms the angle ψ with axis x. From here on, we will signify this orientation by a, so that

$$a=x\cos\psi+y\sin\psi$$

The integration constants $\varepsilon_\perp{}^0$, $\varepsilon_\parallel{}^0$, α, r_0 are so chosen that they have respectively the meaning of the average (in respect to period of rotation in a magnetic field and in respect to period of oscillations in the wave field), of the transverse and longitudinal energies of a particle, of the average initial (i.e. initially averaged, and then taken at $t = 0$) rotation phase and of average initial coordinates of the center of the Larmor circle

$$\varepsilon_\perp=\varepsilon_\perp{}^0+\int\frac{d\varepsilon_\perp}{dt}dt,\qquad \varepsilon_\parallel=\varepsilon_\parallel{}^0+\int\frac{d\varepsilon_\parallel}{dt}dt,$$

$$\varphi=\alpha+\int\frac{d\varphi}{dt}dt,\qquad r=r_0+\int\frac{dr}{dt}dt\tag{2.3}$$

Here $\int\ldots dt$ signifies the indefinite integral in respect to time; the subintegral expressions are the right hand part so of Eq. 2.2a; $r_0 = (x_0, y_0, z_0)$.

Taking into account that the plasma is heterogeneous only in the direction a, and assuming the independence of the equilibrium distribution function from the phase of the Larmor motion of particles, we will have the following dependence of a complete (i.e. taking into account the particles' motion in fields E, B) distribution function f from the motion constants:

$$f(\mathbf{r},\mathbf{v},t)=f_0(\varepsilon_\perp{}^0,\varepsilon_\parallel{}^0,a_0)\tag{2.4}$$

($a_0 = x_0\cos\psi + y_0\sin\psi$; f_0 is a certain function typifying the plasma's state, having the sense of the distribution function of particles for a homogeneous plasma, and in a consideration of the heterogeneity—of the distribution function of the Larmor circles).

We will assume the fields E, B are sufficiently small and we will make use of the method of successive approximations, finding initially the "unperturbed" part of the solution of system 2.2a, and then the "perturbed" part, linear in respect to E. B. The gradient of the constant magnetic field B^0 and the fields g and E^0 are assumed to be small and in the integration of the motion equations of the particles are considered constant.

In this case, the solution of an "unperturbed" system 2.2a with an accuracy up to the first degrees of g, E^0, and ∇B^0 is well known; it corresponds to the Larmor rotation of a particle around the guiding center, slowly displacing across the magnetic field and the direction of the forces, and may be represented in the form:

$$\varepsilon_\perp = \varepsilon_\perp{}^0 - m u_2 v_\perp{}^0 \sin \beta_0$$

$$\varepsilon_\parallel = \varepsilon_\parallel{}^0$$

$$\varphi = \alpha - \omega_c{}^0 t - \frac{u_2 + 2u_1}{v_\perp{}^0} \cos \beta_0 \equiv \varphi_0 - \frac{u_2 + 2u_1}{v_\perp{}^0} \cos \beta_0$$

$$x = x_0 - \varrho_0 \sin \varphi_c + t(u_1 + u_2) \sin \psi + \frac{u_1}{2\omega_0{}^0} \cos(\beta_0 + \varphi_0)$$

$$z = z_0 + v_\parallel{}^0 t; \qquad a = a_0 - \varrho_0 \sin \beta_0 \qquad (2.5)$$

Here

$$u_1 = -\frac{\varrho_0{}^2}{2}\left(\frac{\partial \omega_c}{\partial a}\right)_{a=a_0}, \qquad u_2 = \frac{1}{\omega_c{}^0}\left(g + \frac{eE^0}{m}\right)_\perp,$$

$$\varrho_0 = v_\perp{}^0/\omega_c{}^0, \quad \omega_c{}^0 = \omega_c(a_0),$$

$$\beta_0 = \varphi_0 - \psi, \quad v_\perp{}^0 = (2\varepsilon_\perp{}^0/m)^{1/2}, \quad v_\parallel{}^0 = (2\varepsilon_\parallel{}^0/m)^{1/2} \quad (2.6)$$

The corrections to the undisturbed transverse and longitudinal energies of the particles, which from now on will be symbolized as $\varepsilon_\perp{}'$, $\varepsilon_\parallel{}'$, can be represented in the form:

$$\varepsilon_\perp{}' = \varepsilon_{\perp E} + \varepsilon_B + \varepsilon_F{}',$$
$$\varepsilon_\parallel{}' = \varepsilon_{\parallel E} - \varepsilon_B \qquad (2.7)$$

where

$$\varepsilon_{\perp E} = e\int \mathbf{E}\cdot\mathbf{v}_\perp \, dt$$

$$\varepsilon_{\parallel E} = e\int \mathbf{E}\cdot\mathbf{v}_\parallel \, dt$$

$$\varepsilon_B = \frac{e}{c}\int \mathbf{B}\cdot\mathbf{v}_\perp \times \mathbf{v}_\parallel \, dt \qquad (2.8)$$

$$\varepsilon_F{}' = \left(\int \mathbf{F}\cdot\mathbf{v}_\perp \, dt\right)_B$$

where the subscript "B" signifies "perturbed".

In the integration of the expressions containing E and B, consideration is given to the dependence of the wave amplitude upon a, so that, e.g.

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0(a) \exp(-i\,\omega t + i\,k_x x + i\,k_z z)$$

Generally speaking, it is necessary to take into account the dependence of the wave amplitude upon the coordinates in the case of a heterogeneous plasma, since here (in distinction from the homogeneous plasma) the Maxwell equations possibly can not have solutions in the form of plane waves.

Using Eq. 2.5,

$$E_0(a) = E_0(a_0) - \varrho_0 \sin \beta_0 \frac{\partial E_0}{\partial a}$$

$$\exp(-i\omega t + i k_x x + i k_z z) = \left[1 + \frac{i k_x u_1}{2\omega_c{}^0} \cos(\varphi_0 + \beta_0)\right]$$

$$\times \exp\{i k_x x_0 + i k_z z_0 - i[\omega - k_z v_\parallel{}^0 - k_x(u_1 + u_2)\sin\psi] t$$
$$- i k_x \varrho_0 \sin \varphi_0\} \qquad (2.9)$$

we find the explicit form of the dependence on time of the fields E and B, and analogously for B.

In the integration in the right-hand parts of Eq. 2.8, we will represent the exponent exp $(-i\,k_x\,\varrho_0\sin\varphi_0)$ in the form:

$$\exp(-\mathrm{i}\,k_x\,\varrho_0\sin\varphi_0)=\sum_{n=-\infty}^{+\infty}J_n(k_x\,\varrho_0)\,\mathrm{e}^{-\mathrm{i}n\varphi_0},$$

(J_n = the Bessel function.)

Then with the aid of Eqs. 2.5, 2.8, and 2.9, we find the values $\varepsilon_{\perp}E$, $\varepsilon_{\parallel}E$, εB, $\varepsilon F'$, as the functions r_0, v_0:

$$\begin{aligned}\varepsilon_{\perp E}(\mathbf{r}_0,\mathbf{v}^0)=\mathrm{i}e v_{\perp}^0\sum_n\eta_n^0\mathrm{e}^{-\mathrm{i}n\varphi_0}\Big\{&\frac{nJ_n}{\xi_0}E_x+\mathrm{i}J_n'E_y\\&+\varrho_0\frac{\partial E_x}{\partial a}\Big[(J_n+J_n'')\sin\psi-\mathrm{i}n\Big(\frac{J_n}{\xi_0}\Big)'\cos\psi\Big]\\&+\varrho_0\frac{\partial E_y}{\partial a}\Big[J_n''\cos\psi+\mathrm{i}n\Big(\frac{J_n}{\xi_0}\Big)'\sin\psi\Big]\\&+\frac{u_2}{v_{\perp}^0}J_n(E_x\sin\psi-E_y\cos\psi)\\&+\frac{u_1}{v_{\perp}^0}\Big[E_x\Big\{-\sin\psi\Big[2J_n''+\xi_0\Big(J_n'+\frac{J_n'''}{2}\Big)\Big]\\&\qquad+\frac{\mathrm{i}n}{2}(J_n+2J_n'')\cos\psi\Big\}\\&-E_y\Big\{\Big[\frac{\xi_0}{2}J_n'-\xi_0\Big(1-\frac{n^2}{\xi_0^2}\Big)\Big(\frac{J_n}{\xi_0}\Big)'\Big]\cos\psi+\mathrm{i}nJ_n''\sin\psi\Big\}\Big]\Big\}\\&\times\exp(-\mathrm{i}\,\omega' t+\mathrm{i}\,k_x x_0+\mathrm{i}\,k_z z_0)\end{aligned}\tag{2.10}$$

$$\begin{aligned}\varepsilon_{\parallel E}(\mathbf{r}_0,\mathbf{v}^0)=\mathrm{i}\,e\,v_{\parallel}^0\sum_n\eta_n^0\mathrm{e}^{-\mathrm{i}n\varphi_0}\Big\{&J_nE_z-\mathrm{i}\varrho_0\Big(J_n'\cos\psi+\mathrm{i}\frac{nJ_n}{\xi_0}\sin\psi\Big)\frac{\partial E_z}{\partial a}+\mathrm{i}\frac{u_1}{v_{\perp}^0}\xi_0E_z\Big[\Big(\frac{J_n}{2}+J_n''\Big)\cos\psi\\&+\mathrm{i}\,n\Big(\frac{J_n}{\xi_0}\Big)'\sin\psi\Big]\Big\}\exp(-\mathrm{i}\,\omega' t+\mathrm{i}\,k_x x_0+\mathrm{i}\,k_z z_0)\end{aligned}\tag{2.11}$$

$$\varepsilon_B=\varepsilon_{\perp E}\Big(-\frac{v_{\parallel}^0}{c}B_y,\ \frac{v_{\parallel}^0}{c}B_x\Big)\tag{2.12}$$

$$\begin{aligned}\varepsilon_{F'}(\mathbf{r}_0,\mathbf{v}^0)=-e\,u_2\sum_n\eta_n^0\mathrm{e}^{-\mathrm{i}n\varphi_0}\Big\{&\Big(E_x-\frac{v_{\parallel}^0}{c}B_y\Big)\Big(\mathrm{i}J_n\sin\psi+J_n'\cos\beta_0+\mathrm{i}\frac{nJ_n}{\xi_0}\sin\beta_0\Big)+\mathrm{i}\Big(E_y+\frac{v_{\parallel}^0}{c}B_x\Big)\Big(-J_n\cos\psi\\&+\frac{nJ_n}{\xi_0}\cos\beta_0+\mathrm{i}J_n'\sin\beta_0\Big)+\mathrm{i}\frac{v_{\perp}^0}{c}B_z\Big(\frac{nJ_n}{\xi_0}\cos\psi\\&+\mathrm{i}J_n'\sin\psi-J_n\cos\beta_0\Big)\Big\}\exp(-\mathrm{i}\,\omega' t+\mathrm{i}\,k_x x_0+\mathrm{i}\,k_z z_0)\end{aligned}\tag{2.13}$$

where

$$\xi_0=\varrho_0 k_x,\quad \eta_n^0=(\omega'-n\,\omega_c^0)^{-1},$$

$$\omega'=\omega-k_z v_{\parallel}^0-k_x(u_1+u_2)\sin\psi\,.$$

It is also necessary to compute an expression for the disturbed part of the shift of a particle along the direction of the gradient. $a' = x' \cos\psi + y' \sin\psi$. It is easy to see that a' has a form analogous to ε_F' (Eq. 2.13), only in it in place of F, there is the unit:

$$a' = \frac{1}{F}\,\varepsilon_F'\,. \qquad (2.14)$$

Hence we have found a solved for the system of 2.2. In order to write a solution of Eq. 2.1, it is necessary to express the motion constants by r, v, t. Using Eqs. 2.4 – 2.14, we find

$$\begin{aligned}
\varepsilon_\perp{}^0(\mathbf{r},\mathbf{v},t) &= \varepsilon_\perp - \varepsilon_\perp{}'' - \varepsilon_\perp{}'(\mathbf{r},\mathbf{v},t)\\
\varepsilon_\parallel{}^0(\mathbf{r},\mathbf{v},t) &= \varepsilon_\parallel - \varepsilon_\parallel{}'(\mathbf{r},\mathbf{v},t) \qquad (2.15)\\
a_0(\mathbf{r},\mathbf{v},t) &= a - a'' - a'(\mathbf{r},\mathbf{v},t)
\end{aligned}$$

where

$$\begin{aligned}
&\varepsilon_\perp{}'' = -m\,u_2\,v_\perp \sin\beta\,, \qquad a = -\varrho\sin\beta\,,\\
&\varepsilon_\perp{}' = \varepsilon_{\perp E}(\mathbf{r},\mathbf{v},t) + \varepsilon_F'(\mathbf{r},\mathbf{v},t) + \varepsilon_B(\mathbf{r},\mathbf{v},t) \qquad (2.16)\\
&\varepsilon_\parallel{}' = \varepsilon_{\parallel E}(\mathbf{r},\mathbf{v},t) - \varepsilon_{\parallel}(\mathbf{r},\mathbf{v},t)
\end{aligned}$$

The expressions $\varepsilon_\perp E(\mathbf{r},\mathbf{v})$ and $\varepsilon_\parallel(\mathbf{r},\mathbf{v})$ have the form:

$$\varepsilon_{\perp E}(\mathbf{r},\mathbf{v}) = \mathrm{i}\,e\,v_\perp\,e^{\mathrm{i}\xi\sin\varphi}\sum_n\Big(\eta_n\,\mathbf{q}_\perp{}^{(n)}\cdot\mathbf{E}_\perp + \mathbf{q}_{\perp 0}{}^{(n)}\cdot\mathbf{E}_\perp a''\frac{\partial\eta_n}{\partial a}\Big)e^{-\mathrm{i}n\varphi}$$

$$\varepsilon_{\parallel E}(\mathbf{r},\mathbf{v}) = \mathrm{i}\,e\,v_\parallel e^{\mathrm{i}\xi\sin\varphi}\sum_n\Big(\eta_n\,q_z{}^{(n)}\,E_z + q_{z0}{}^{(n)}E_z\,a''\frac{\partial\eta_n}{\partial a}\Big)e^{-\mathrm{i}n\varphi} \qquad (2.17)$$

where

$$\mathbf{q} = \mathbf{q}_0 + \mathbf{q}' \qquad (2.18)$$

$$q_{0x} = n\,J_n/\xi\,;\; q_{0y} = \mathrm{i}\,J_n'\,;\; q_{0z} = J_n \qquad (2.19)$$

$$\begin{aligned}
q_x' = v_x\Big[&(J_n + J_n'')\sin\psi - \mathrm{i}\,n\Big(\frac{J_n}{\xi}\Big)'\cos\psi\\
&+ \frac{nJ_n}{\xi}\sin\beta\Big] + \frac{u_2}{v_\perp}\,J_n\sin\psi + n\,J_n'\sin\beta\\
&+ \mathrm{i}\,nJ_n\Big(\cos\psi - \frac{n}{\xi}\cos\beta\Big)\Big] + \frac{u_\perp}{v_\perp}\Big[2n\xi\Big(\frac{J_n}{\xi}\Big)'\sin\beta\\
&+ 2\,\mathrm{i}\,nJ_n\Big(\cos\psi - \frac{n}{\xi}\cos\beta\Big) + \mathrm{i}\,n\cos\psi\Big(\frac{J_n}{2}\\
&+ J_n''\Big) - \frac{\mathrm{i}n}{2}\,J_n\cos(\varphi+\beta) - 2\,J_n''\sin\psi\\
&- \xi\Big(J_n' + \frac{J_n'''}{2}\Big)\sin\psi\Big] \qquad (2.20)
\end{aligned}$$

$$
\begin{aligned}
q_y' = v_y \Big[& \mathrm{i}\, n \left(\frac{J_n}{\xi}\right)' \sin\psi + J_n'' \cos\psi + \mathrm{i} J_n' \sin\beta \Big] \\
& - \frac{u_2}{v_\perp} \left(J_n \cos\psi - \mathrm{i} J_n' \sin\beta \right) \\
& - \frac{2u_1 + u_2}{2 v_\perp} \xi \Big[J_n' \left(\cos\psi - \frac{n}{\xi} \cos\beta \right) - \mathrm{i} J_n'' \sin\beta \Big] \\
& - \frac{u_1}{v_\perp} \Big[\cos\psi \left(J_n + n^2 \left(\frac{J_n}{\xi}\right)' - \frac{\xi}{2} J_n' (\cos\psi \right. \\
& \left. + \cos(\varphi + \beta) \right) + \mathrm{i}\, n \sin\psi\, J_n'' \Big]
\end{aligned}
\tag{2.20}
$$

$$
\begin{aligned}
q_z' = v_z \Big[& \frac{n J_n}{\xi} \sin\psi - \mathrm{i} J_n' \cos\psi + J_n \sin\beta \Big] \\
& + \frac{2u_1 + u_2}{v_\perp} \xi \Big[J_n' \sin\beta + \mathrm{i} J_n \left(\cos\psi - \frac{n}{\xi} \cos\beta \right) \Big] \\
& + \frac{\mathrm{i} u_1}{v_\perp} \xi \Big[\left(\frac{J_n}{2} + J_n'' \right) \cos\psi + \mathrm{i}\, n \left(\frac{J_n}{\xi} \right)' \sin\psi \\
& - \frac{J_n}{2} \cos(\varphi + \beta) \Big],
\end{aligned}
$$

$$
\nu = \frac{\varrho}{E_0} \frac{\partial E_0}{\partial a}, \quad \xi = k_x v_\perp / \omega_c, \quad \varrho = v_\perp / \omega_c,
$$

$$
\eta_n = (\omega' - n\,\omega_c)^{-1}
$$

The expression $\varepsilon_{F'}$ has the form:

$$
\begin{aligned}
\varepsilon_F'(\mathbf{r}, \mathbf{v}) = -\mathrm{i} e\, u_2\, e^{\mathrm{i}\xi \sin\varphi} \sum_n \eta_n\, e^{-\mathrm{i} n \varphi} \Big\{ & \Big(E_x \\
& - \frac{v_z}{c} B_y \Big) J_n \sin\psi - \left(E_y + \frac{v_z}{c} B_x \right) J_n \cos\psi \\
& + \mathrm{i} \frac{v_\perp}{c} B_z \left(J_n' \sin\psi - \mathrm{i} \frac{n J_n}{\xi} \cos\psi \right) \Big\}
\end{aligned}
\tag{2.21}
$$

The relationships in Eqs. 2.12 and 2.14, linking ε_B with $\varepsilon_{\perp E}$ and respectively a' with $\varepsilon_{F'}$, retain their former appearance.

Taking Eq. 2.4 into account, we derive:

$$
f(\mathbf{r}, \mathbf{v}, t) = f_0 \left[\varepsilon_\perp - \varepsilon_\perp'' - \varepsilon_\perp'(\mathbf{r}, \mathbf{v}),\ \varepsilon_\parallel - \varepsilon_\parallel'(\mathbf{r}, \mathbf{v}),\ a - a'' - a'(\mathbf{r}, \mathbf{v}) \right]
\tag{2.22}
$$

Considering $\varepsilon_\perp''$, $\varepsilon_\perp'$, $\varepsilon_\parallel'$, a'', and a' to be small values, we can conduct an expansion of the right part of Eq. 2.22 into a series with respect to these values. Then the terms not containing the perturbations will yield the equilibrium distribution function, f^0, equalling

$$
f^0(\mathbf{r}, \mathbf{v}) = f_0(\varepsilon_\parallel, \varepsilon_\perp, a) - \frac{\partial f_0}{\partial \varepsilon_\perp} \varepsilon_\perp'' - \frac{\partial f_0}{\partial a} a''
\tag{2.23}
$$

while the terms containing the fields E, B, will yield the perturbed function, f^1, so that

$$
\begin{aligned}
f^1(\mathbf{r}, \mathbf{v}, t) = - \Big[& (\varepsilon_{\perp E} + \varepsilon_{F'}) \frac{\partial f_0}{\partial \varepsilon_\perp} + \varepsilon_{\parallel E} \frac{\partial f_0}{\partial \varepsilon_\parallel} \\
& - \varepsilon_B \left(\frac{\partial f_0}{\partial \varepsilon_\perp} - \frac{\partial f_0}{\partial \varepsilon_\parallel} \right) - a' \frac{\partial f_0}{\partial a} \\
& + a'' \Big[(\varepsilon_{\perp E} + \varepsilon_{F'}) \frac{\partial^2 f_0}{\partial \varepsilon_\perp \partial a} + \varepsilon_{\parallel E} \frac{\partial^2 f_0}{\partial \varepsilon_\parallel \partial a} \\
& + a'' \varepsilon_B \left(\frac{\partial^2 f_0}{\partial \varepsilon_\perp \partial a} - \frac{\partial^2 f_0}{\partial \varepsilon_\parallel \partial a} \right)
\end{aligned}
\tag{2.24}
$$

$$+\varepsilon_{\perp}{}^{*}\left(\varepsilon_{\perp}E\frac{\partial^2 f_0}{\partial\varepsilon_{\perp}{}^2}+\varepsilon E\frac{\partial^2 f_0}{\partial\varepsilon_{\perp}\partial\varepsilon_z}\right)$$

$$+\varepsilon_{\perp}{}^{*}\,\varepsilon E\left(\frac{\partial^2 f_0}{\partial\varepsilon_{\perp}{}^2}-\frac{\partial^2 f_0}{\partial\varepsilon_{\perp}\partial\varepsilon_z}\right) \qquad (2.23, \text{ cont'd})$$

Having expressed in the right hand part of Eq. 2.24 B by E (with the aid of the equation $\partial B/\partial t = -c\nabla \times E$), we get:

$$f^1=-i\,e\sum_{n=-\infty}^{\infty}e^{i\xi\sin\varphi-in\varphi}\ldots\{v_{\perp}q_{\perp}\cdot E_{\perp}(\Psi_{\perp}$$

$$+\varrho\Phi_{\perp}\sin\beta)+v_z q_z E_z(\Psi_z+\varrho\Phi_z\sin\beta)+\tau\cdot E\Phi_a\} \qquad (2.25)$$

where

$$\Psi_{\perp}=\frac{\partial f_0}{\partial\varepsilon_{\perp}}-\frac{v_z k_z}{\omega}\left(\frac{\partial f_0}{\partial\varepsilon_{\perp}}-\frac{\partial f_0}{\partial\varepsilon_z}\right)$$

$$\Psi_z=\frac{\partial f_0}{\partial\varepsilon_z}+\frac{n\omega_c+k_x(u_{\perp}-u_z)\sin\gamma}{\omega}\left(\frac{\partial f_0}{\partial\varepsilon_{\perp}}-\frac{\partial f_0}{\partial\varepsilon_z}\right)$$

$$\Phi_{\perp}=\left(\frac{\partial}{\partial a}+P\frac{\partial}{\partial\varepsilon_{\perp}}+\frac{2n\,u_1}{\varrho^2}\,\eta_n\right)\Psi_{\perp}$$

$$\Phi_z=\left(\frac{\partial}{\partial a}+P\frac{\partial}{\partial\varepsilon_{\perp}}\frac{2n\,u_1}{\varrho^2}\eta_n\right)\Psi_z+\frac{2n\,u_1}{\omega\varrho^2}\left(\frac{\partial f_0}{\partial\varepsilon_{\perp}}-\frac{\partial f_0}{\partial\varepsilon_z}\right)$$

$$\Phi_a=\left(\frac{\partial}{\partial a}+P\frac{\partial}{\partial\varepsilon_{\perp}}\right)f_0 \qquad (2.26)$$

The vector τ has the components:

$$\tau_x=-\frac{J_n\sin\psi}{m\,\omega_c}\left(1-\frac{v_z k_z}{\omega}\right)$$

$$\tau_y=\frac{1}{m\,\omega_c\,\omega}[(\omega-n\,\omega_c-k_z v_z)\,J_n\cos\psi$$

$$-i\,v_{\perp}\,k_x\,J_n'\sin\psi] \qquad (2.27)$$

$$\tau_z=-\frac{v_z k_x J_n\ldots\psi}{m\,\omega_c\,\omega}$$

3 Flows in a Plasma

With the aid of the distribution function f^1 found in Sect. 2, we can compute the density of the currents induced by an emf in the plasma.

We will now write, omitting the intermediate steps, the expressions for the density of currents and density of the charges.

The current linked with each type of charge has the form:

$$j_\alpha=\sigma_{\alpha\beta}\,E_\beta \qquad (3.1)$$

where

$$\sigma_{\alpha\beta}(\alpha,\beta=1,2)=-ie^2\sum_{n=-\infty}^{\infty}\int d\varepsilon_{\perp}\,dv_z\,\eta_n\,v_{\perp}{}^2\,(q_{\alpha\beta}\Psi_{\perp}$$

$$+\varrho\,\mu_{\alpha\beta}\,\Phi_{\perp}+\tau_{\alpha\beta}\,\Phi_a) \qquad (3.2)$$

(Eq. cont'd, next page)

$$\sigma_{3\beta}(\beta=1,2)=-ie^2\sum_{n=-\infty}^{\infty}\int d\varepsilon_\perp\, dv_z\, \eta_n v_\perp v_z (q_{3\beta}\Psi_n + g\,\mu_{3\beta}\Phi_n + \tau_{3\beta}\Phi_z) \qquad (3.2)$$

$$\sigma_{\alpha 3}(\alpha=1,2)=-ie^2\sum_{n=-\infty}^{\infty}\int d\varepsilon_\perp\, dv_z\, \eta_n v_z v_\perp (q_{\alpha 3}\Psi_n + g\,\mu_{\alpha 3}\Phi_n + \tau_{\alpha 3}\Phi_z)$$

$$\sigma_{33}=-ie^2\sum_{n=-\infty}^{\infty}\int d\varepsilon_\perp\, dv_z\, \eta_n v_z^2 (q_{33}\Psi_n + g\,\mu_{33}\Phi_n + \tau_{33}\Phi_z)$$

The components of the tensor $q_{\alpha\beta}$ are linked with the vector (of) q in the following manner:

$$q_{\alpha\beta}=\begin{pmatrix} \langle q_x\cos\varphi\rangle & \langle q_y\cos\varphi\rangle & \langle q_z\cos\varphi\rangle \\ \langle q_x\sin\varphi\rangle & \langle q_y\sin\varphi\rangle & \langle q_z\sin\varphi\rangle \\ \langle q_x\rangle & \langle q_y\rangle & \langle q_z\rangle \end{pmatrix} \qquad (3.3)$$

The bracket $\langle\rangle$ around the A-value signifies

$$\langle A\rangle=\frac{1}{2\pi}\int_0^{2\pi} A(\varphi)\, e^{i\xi\sin\varphi-in\varphi}\, d\varphi \qquad (3.4)$$

As is evident from Eqs. 2.24 and 2.25, the vector q consists of four groups of terms, one of which does not depend on the heterogeneities of the fields or the forces F, the other is attributable to the heterogeneity of the wave amplitude, the third to the force F, and the last one—to the gradient of the constant magnetic field B^0. In conformity with this, we represent the tensor $q_{\alpha\beta}$ in the form:

$$q_{\alpha\beta}=q_{\alpha\beta}^0+v_{\alpha\beta}+\frac{u_2}{v_\perp}G_{\alpha\beta}+\frac{u_1}{v_\perp}H_{\alpha\beta}; \qquad (3.5)$$

u_2 and u_1 = the velocities of the gravitational (and electrical) and diamagnetic drifts (see Eq. 2.6).

Here $q_{\alpha\beta}{}^0$ = the part of the tensor $q_{\alpha\beta}$, not connected with the heterogeneity:

$$q_{\alpha\beta}^0=q_{\alpha 0}^*\, q_{\beta 0}=\begin{pmatrix} \frac{n^2 J_n^2}{\xi^2} & \frac{i\,n J_n J_n'}{\xi} & \frac{n J_n^2}{\xi} \\ \frac{-i n J_n J_n'}{\xi} & J_n'^2 & -i J_n J_n' \\ \frac{n J_n^2}{\xi} & i J_n J_n' & J_n^2 \end{pmatrix} \qquad (3.6)$$

The tensor $v_{\alpha\beta}$ is connected with the heterogeneity of the wave amplitude, and as follows from Eq. 2.25, has the form:

$$\nu_{\alpha\beta} = \nu_{\alpha\beta}' \sin\psi + \nu_{\alpha\beta}'' \cos\psi \tag{3.7}$$

where

$$\nu_{\alpha\beta}' = -\frac{1}{\xi} \begin{pmatrix} 0 & (q_{11}^0 - q_{22}^0)\,\nu_y & -q_{23}^0\,\nu_z \\ (q_{11}^0 - q_{22}^0)\,\nu_x & 0 & q_{13}^0\,\nu_z \\ q_{32}^0\,\nu_x & q_{31}^0\,\nu_y & 0 \end{pmatrix} \tag{3.8}$$

$$\nu_{\alpha\beta}'' = -i\frac{\partial}{\partial\xi} \begin{pmatrix} q_{11}^0\,\nu_x & q_{12}^0\,\nu_y & q_{13}^0\,\nu_z \\ q_{21}^0\,\nu_x & q_{22}^0\,\nu_y & q_{23}^0\,\nu_z \\ q_{31}^0\,\nu_x & q_{32}^0\,\nu_y & q_{33}^0\,\nu_z \end{pmatrix} \tag{3.9}$$

The tensor $G_{\alpha\beta}$, linked with the effect of force F, has the appearance as in Eq. 3.10.

$$G_{11} = \sin\psi \left\{ \frac{nJ_n^2}{\xi} - n\left[J_n'(J_n + J_n'') + \frac{n^2 J_n}{\xi}\left(\frac{J_n}{\xi}\right)' \right] \right\} - i\,n^2 \left(\frac{J_n J_n'}{\xi}\right)' \cos\psi$$

$$G_{12} = i\sin\psi \left\{ (J_n + J_n'')(J_n' - \xi J_n'') + n^2 J_n'\left(\frac{J_n}{\xi}\right)' \right\} + n\cos\psi \left[\frac{J_n^2}{\xi} - J_n'\left(\frac{J_n}{\xi}\right)' + J_n''\left(2J_n' - \frac{J_n}{\xi}\right) \right]$$

$$G_{21} = -i\sin\psi \left\{ J_n J_n' - n^2\left(\frac{J_n J_n'}{\xi}\right)' \right\} - \frac{n\cos\psi}{\xi}\left[J_n'^2 + 2\xi J_n J_n' - \frac{n^2 J_n^2}{\xi} \right]$$

$$G_{22} = -n\sin\psi \left[J_n''\left(2J_n' - \frac{J_n}{\xi}\right) + J_n'\left(\frac{J_n}{\xi}\right)' \right] + i\cos\psi\left[\xi J_n'^2 - n^2 J_n'\left(\frac{J_n}{\xi}\right)' - \xi J_n''^2 - J_n' J_n'' \right]$$

$$G_{13} = \sin\psi \left[J_n'^2 + \frac{n^2 J_n^2}{\xi^2} - \frac{2n^2 J_n J_n'}{\xi} \right] - i\,n\cos\psi\,\xi\left(\frac{J_n J_n'}{\xi}\right)'$$

$$G_{31} = \sin\psi \left[J_n^2 - \frac{2n^2}{\xi} J_n J_n' \right] + i\,n\cos\psi \left[J_n^2\left(1 - \frac{n^2}{\xi^2}\right) - J_n'^2 \right]$$

$$G_{23} = i\,n\,\xi\sin\psi\left(\frac{J_n J_n'}{\xi}\right)' - \cos\psi\left[J_n'^2 + 2\xi J_n J_n' - \frac{n^2 J_n^2}{\xi^2} \right]$$

$$G_{32} = i\,n\sin\psi\left[J_n^2\left(1 - \frac{n^2}{\xi^2}\right) - J_n'^2 \right] - \cos\psi\, J_n\left[J_n + 2\xi J_n'\left(1 - \frac{n^2}{\xi^2}\right) \right]$$

$$G_{33} = -2n\, J_n J_n' \sin\psi + i\,\xi\left[J_n^2\left(1 - \frac{n^2}{\xi^2}\right) - J_n'^2 \right]\cos\psi \tag{3.10}$$

$$H_{11}=2n\sin\psi\left[J_n'\left(\frac{J_n}{\xi}\right)'-\frac{J_nJ_n''}{\xi}-J_n\left(J_n'+\frac{J_n'''}{2}\right)\right]-\frac{2in^2}{\xi^2}\cos\psi\left[\xi(J_n')^2-3J_nJ_n'+\frac{2J_n^2}{\xi}+\xi J_nJ_n''\right]$$

$$H_{12}=i\sin\psi\left\{-2\xi J_n''(J_n+J_n'')+n^2\left[2J_n'\left(\frac{J_n}{\xi}\right)'+\frac{J_nJ_n''}{\xi}-\xi J_n'^2-\frac{\xi}{2}J_n'J_n'''\right]\right\}-n\cos\psi\left[\frac{J_n^2}{\xi}\right.$$
$$\left.-2J_n''\left(2J_n'-\frac{J_n}{\xi}\right)+\frac{n^2}{\xi}J_n\left(\frac{J_n}{\xi}\right)'-J_nJ_n'-\xi J_n'\left(\frac{J_n}{\xi}\right)''\right]$$

$$H_{21}=-i\sin\psi\left[2n^2J_n'\left(\frac{J_n}{\xi}\right)'+\frac{n^2J_nJ_n''}{\xi}+2J_n'J_n''+\xi\left(J_n'+\frac{J_n'''}{2}\right)\right]+n\cos\psi\left[3J_nJ_n'-2\xi J_n''\left(\frac{J_n}{\xi}\right)'\right.$$
$$\left.-\frac{n^2J_n}{\xi}\left(\frac{J_n}{\xi}\right)'+J_n'J_n''+\frac{J_nJ_n'''}{2}\right]$$

$$H_{22}=2n\sin\psi\left[J_n'\left(\frac{J_n}{\xi}\right)'+J_n''\left(\frac{J_n}{\xi}-3J_n'\right)\right]+i\cos\psi\left[J_n'\left(J_n-\frac{\xi}{2}J_n'''\right)+\xi J_n'^2-n^2J_n'\left(\frac{J_n}{\xi}\right)'-2\xi J_n''^2\right]$$

$$H_{13}=-\xi\sin\psi\left[J_n'(3J_n+2J_n'')+\frac{3n^2J_n}{\xi}\left(\frac{J_n}{\xi}\right)'+\frac{1}{2}J_nJ_n'''\right]+2in\cos\psi\left[\frac{J_n^2}{\xi^2}-(J_nJ_n')'\right]$$

$$H_{31}=-i\sin\psi J_n\left[3n^2\left(\frac{J_n}{\xi}\right)'+2n^2\frac{J_n'}{\xi}+2J_n''+\xi\left(J_n'+\frac{J_n'''}{\xi}\right)\right]+2in\cos\psi\left[\xi J_n'\left(\frac{J_n}{\xi}\right)'+J_n''-\frac{n^2J_n^2}{\xi^2}\right]$$

$$H_{23}=in\xi\sin\psi\left\{2\left(\frac{J_nJ_n'}{\xi}\right)'+\left[J_n\left(\frac{J_n}{\xi}\right)'\right]'\right\}-\xi\cos\psi\left[3J_n'J_n''-2J_nJ_n'+\frac{2n^2J_n}{\xi}\left(\frac{J_n}{\xi}\right)'-\frac{1}{2}J_nJ_n'''\right]$$

$$H_{32}=in\sin\psi\left[\frac{J_nJ_n'}{\xi}-\xi(J_nJ_n')'\right]-\cos\psi\left[3J_n'^2+J_n\left(1-\frac{n^2}{\xi^2}\right)(J_n+5\xi J_n')\right]$$

$$H_{33}=2nJ_n\left[\frac{J_n}{\xi}-3J_n'\right]\sin\psi+2i\xi\left[J_n^2\left(1-\frac{n^2}{\xi^2}\right)-J_n'^2\right]\cos\psi \qquad (3.11)$$

$$\mu_{\alpha\beta}=\begin{pmatrix}-nJ_n(J_n+J_n'') & -iJ_n'(J_n+J_n'') & -J_n(J_n+J_n'')\\ \frac{in^2J_n}{\xi}\left(\frac{J_n}{\xi}\right)' & -nJ_n'\left(\frac{J_n}{\xi}\right)' & inJ_n\left(\frac{J_n}{\xi}\right)'\\ -\frac{n^2J_n^2}{\xi} & -in\frac{J_nJ_n'}{\xi} & -\frac{nJ_n^2}{\xi}\end{pmatrix}\sin\psi$$
$$+\begin{pmatrix}-\frac{in^2}{\xi}J_n\left(\frac{J_n}{\xi}\right)' & nJ_n'\left(\frac{J_n}{\xi}\right)' & -inJ_n\left(\frac{J_n}{\xi}\right)'\\ -\frac{nJ_nJ_n''}{\xi} & -iJ_n'J_n'' & -J_nJ_n''\\ -\frac{inJ_nJ_n'}{\xi} & J_n'^2 & -iJ_nJ_n'\end{pmatrix}\cos\psi \qquad (3.14)$$

$$\tau_{\alpha\beta} = -\frac{\sin\psi}{m v_\perp \omega_c} \begin{pmatrix} \frac{n J_n^2}{\xi}\left(1-\frac{v_\parallel k_z}{\omega}\right) & i\frac{\omega_c}{\omega} n J_n J_n' & \frac{\omega_c}{\omega} n J_n^2 \\ -iJ_n J_n'\left(1-\frac{v_\parallel k_z}{\omega}\right) & \frac{\omega_c}{\omega}\xi J_n'^2 & -i\frac{\omega_c}{\omega}\xi J_n J_n' \\ J_n\left(1-\frac{v_\parallel k_z}{\omega}\right) & i\frac{\omega_c}{\omega}\xi J_n J_n' & \frac{\omega_c}{\omega}\xi J_n^2 \end{pmatrix}$$

$$+\frac{\cos\psi}{m v_\perp \omega_c}\left(1-\frac{n\omega_c + k_z v_\parallel}{\omega}\right)\begin{pmatrix} 0 & n J_n^2/\xi & 0 \\ 0 & -iJ_nJ_n' & 0 \\ 0 & J_n^2 & 0 \end{pmatrix} \qquad (3.16)$$

The tensor $H_{\alpha\beta}$, conditioned by the heterogeneity of the magnetic field, has the appearance as shown in Eq. 3.11.

The tensor $\mu_{\alpha\beta}$ is determined by the relationship

$$\mu_{\alpha\beta} = b_\alpha g_\beta^0 \qquad (3.12)$$

where

$$b_1 = \langle \cos\varphi \sin\beta \rangle, \quad b_2 = \langle \sin\varphi \sin\beta \rangle, \quad b_3 = \langle \sin\beta \rangle \qquad (3.13)$$

In this manner, $\mu_{\alpha\beta}$ has the form as in Eq. 3.14.

Finally, the last of the tensors contained in the integrand of Eq. 3.2, $\tau_{\alpha\beta}$ is determined in the following manner:

$$\tau_{\alpha\beta} = \frac{1}{v_\perp} q_\alpha^{0*} \tau_\beta \quad \alpha=1,2,3; \quad \beta=1,2$$

$$\tau_{\alpha 3} = \frac{1}{v_\parallel} q_\alpha^{0*} \tau_3 \quad \alpha=1,2,3; \quad \beta=3 \qquad (3.15)$$

and from this Eq. 3.16 is derived.

The relationships of Eqs. 3.1 – 3.16 together with Eq. 2.33 completely determine the currents being induced by an electromagnetic field in a heterogeneous plasma, as the functions of the equilibrium characteristics of the plasma, of the amplitudes of the wave's electric field, and of the derivatives of the amplitude in the direction of the plasma's heterogeneity.

Substituting j, expressed by E, into the Maxwell equation:

$$\nabla\times\nabla\times \mathbf{E} - \frac{\omega^2}{c^2}\mathbf{E} = \frac{4\pi i\omega}{c^2}\mathbf{j} \qquad (3.17)$$

we derive a system of equations into which only the amplitudes of the wave's electrical field are unknowns.

We note that in an examination of the longitudinal oscillations, ie. those in which $\nabla \times E \approx 0$, it is convenient to use in place of Eq. 3.17 the Poisson equation:

$$\nabla\cdot\mathbf{E} = 4\pi\varrho \qquad (3.18)$$

The density of charge ϱ, entering this equation can be expressed by the currents j already found (as functions of the electrical field) with the aid of the continuity equations. However, it is simpler to find ϱ not with

the aid of $\sigma_{\gamma\beta}$ but directly with the aid of the distribution function f^1 (see Eq. 2.25). Having integrated Eq. 2.25 with respect to velocities, we get the following expression for the charge density of each type of particles:

$$\rho = X \cdot A \qquad (3.19)$$

where

$$X_\alpha (\alpha = 1,2) = -i e^2 \sum_{n=-\infty}^{+\infty} \int d\varepsilon_- \, dv_\perp \, \eta_n v_\perp (q_{\alpha 2} \Psi_- + \varrho \mu_{3\alpha} \Phi_\perp + \tau_{3\alpha} \Phi_3)$$

$$X_3 = -i e^2 \sum_{n=-\infty}^{+\infty} \int d\varepsilon_- \, dv_\perp \, \eta_n v_\perp (q_{33} \Psi_- + \varrho \mu_{33} \Phi_- + \tau_{33} \Phi_\alpha) \qquad (3.20)$$

4 Second Approximation in Respect to Gradients for Low-Pressure Plasma

Sometimes for very low frequency waves ($\omega \ll \omega_c$) the terms in proportional to the zero and first degrees of the gradients of the equilibrium parameters may become very small, so that for the study of such waves, it may prove necessary to take into account in the distribution function the terms that are small, i.e. as the square of the gradient. In this section, we solve for the distribution function, taking such small terms into consideration.

We limit ourselves to a case when the plasma pressure is much less than that of the magnetic field (low-pressure plasma). As is known, then the gradient of the equilibrium distribution function can be considered as much larger than the gradient of the magnetic field, and, taking into account the squares of the gradients of the distribution function, we can disregard the values that are quadratic (and of a higher order) in respect to the gradient of the magnetic field. Furthermore, the velocity of the gravitational drift will be regarded as appreciably higher than that of the diamagnetic drift, but much lower than the velocity of the Larmor drift, i.e.

$$\left| \frac{v^2}{2} \frac{\partial \omega_c}{\partial a} \right| \ll \left| \frac{g}{\omega_c} \right| \ll \left| \frac{1}{n_0} \frac{\partial}{\partial a} \int \frac{m v_\perp^2}{2 \omega_c} f_0 \, d\mathbf{v} \right| \qquad (4.1)$$

$$n_0 = \int f_0 \, d\mathbf{v}$$

Then we can also disregard the values of the order g^2, but we retain the values of the order $g \, \partial f_0 / \partial a$. In this manner, the second approximation of the present section signifies making allowance for the terms of the order of $\partial^2 f_0 / \partial a^2$, $g \partial f_0 / \partial a$.

For convenience, we select such a reading system, when in the point in space under consideration, the equilibrium electrical field equals zero, $E^0(a) = 0$. Understandably, in the adjacents points, with some other coordinate a, E^0 generally speaking is not necessarily reverted to zero. However, estimations indicate that the effects linked with the derivatives (of) E^0, are insignificant.

Using the assumptions made, we find that a solution of the system of equations for the unperturbed motion of a particle will as previously be determined by Eq. 2.5, in which however, u_2 = the velocity of only the grav-

itational drift (since $E^0 = 0$). All of the ensuing relationships also retain their former appearance, with the exception of the expressions for the displacement of a particle in the direction of the heterogeneity, which (expressions) should also contain the terms proportional to u_z. Thus to the value $a'(r_c, v^0)$ (see Eq. 2.14) there should be added $\delta a'$, where

$$\delta a'(r_0, v^0, t) = \frac{e u_z B_z}{m \omega_c^2 c} \sum_n \eta_n^0 e^{-in\varphi_0} \left(J_n' \cos\varphi_0 + \frac{in J_n}{\xi_0} \sin\varphi_0 \right) \tag{4.2}$$

The second equation in the system 2.16 will also change and acquire the form:

$$a''(r, v) = -\varrho \sin\beta - \frac{u_z}{\omega_c} \tag{4.3}$$

while in the third equation of system 2.15, the value for $a'(r, v, t)$ is now fixed by the equation

$$\begin{aligned} a'(r, v, t) = &-\frac{ie}{m\omega_c} e^{i\xi\sin\psi} \sum_n \eta_n e^{-in\psi} \Big\{ \Big[\Big(E_x \\ &- \frac{v_z}{c} B_y \Big) \sin\psi - \Big(E_y + \frac{v_z}{c} B_x \Big) \cos\psi \Big] \\ &\times \Big[J_n \Big(1 + i\xi \frac{u_z}{v_\perp} \cos\psi - \frac{i u_z}{v_\perp} n \cos\beta \Big) \\ &+ \frac{u_z}{v_\perp} \xi J_n' \sin\beta \Big] + \frac{v_\perp}{c} B_z \Big[\frac{n}{\xi} \cos\psi \Big[J_n \Big(1 \\ &+ i\xi \frac{u_z}{v_\perp} \cos\psi - \frac{i u_z}{v_\perp} n \cos\beta + \frac{u_z}{v_\perp} \xi J_n' \sin\beta \Big] \\ &+ i \sin\psi \Big[J_n' \Big(1 + i\xi \frac{u_z}{v_\perp} \cos\psi - i \frac{u_z}{v_\perp} n \cos\beta \\ &+ \frac{u_z}{v_\perp} n \cos\beta + \frac{u_z}{v_\perp} \sin\beta \Big) + \frac{u_z}{v_\perp} \xi J_n'' \sin\beta \Big] \Big] \Big\} \end{aligned} \tag{4.4}$$

Finally, to the expression for the disturbed distribution function (see Eq. 2.24) there should be added δf^1, where

$$\begin{aligned} \delta f^1 = &\varepsilon_\perp'' a' \frac{\partial^2 f_0}{\partial \varepsilon_\perp \partial a} - \varepsilon'' a'' \Big(\varepsilon_{\perp E} \frac{\partial^3 f_0}{\partial \varepsilon_\perp^2 \partial a} \\ &+ \varepsilon_{\parallel E} \frac{\partial^3 f_0}{\partial \varepsilon_\perp \partial \varepsilon_\parallel \partial a} \Big) - \varepsilon_\perp'' a'' \varepsilon_B \Big(\frac{\partial^3 f_0}{\partial \varepsilon_\perp^2 \partial a} - \frac{\partial^3 f_0}{\partial \varepsilon_\perp \partial \varepsilon_\parallel \partial a} \Big) \\ &+ a'' a' \frac{\partial^2 f_0}{\partial a^2} - a''^2 \Big(\varepsilon_{\perp E} \frac{\partial^3 f_0}{\partial \varepsilon_\perp \partial a^2} + \varepsilon_{\parallel E} \frac{\partial^3 f_0}{\partial \varepsilon_\parallel \partial a^2} \Big) \\ &- a''^2 \varepsilon_B \Big(\frac{\partial^3 f_0}{\partial \varepsilon_\perp \partial a^2} - \frac{\partial^3 f_0}{\partial \varepsilon_\parallel \partial a^2} \Big) \end{aligned} \tag{4.5}$$

This equation is valid at arbitrary values of ω/ω_c and ξ. However, at $\omega/\omega_c \gtrsim 1$, $\xi \gtrsim 1$, δf^1 constitutes a very slight addition to f^1; also of low value are the corresponding additions to the tensor of conductivity $\sigma_{\alpha\beta}$. We will show that these additions become significant at $\omega \leqslant \omega_c$.

For simplicity, we will assume that the function f_0 is Maxwellian with a temperature independent of a. We shall disregard the gradient of the constant magnetic field, $\nabla B^0 = 0$, and also the coordinate dependence of the variable fields' amplitudes. We consider a wave propagating across the magnetic field ($k_z = 0$) and across a, ($\psi = \pi/2$).

In this case, with the aid of the function $f^1 + \delta f^1$, we derive, e.g., the following expression for σ_{xx}:

$$
\begin{aligned}
\sigma_{xx} = \frac{ie^2 n_0}{m} \sum_{n=-\infty}^{\infty} \eta_n \int f_\perp \, d\varepsilon_\perp \Big\{ & n^2 \frac{m\omega_c^2}{T k_x^2} J_n^2 \\
& + n \frac{\varkappa}{k_x} \left[J_n^2 - \frac{m v_\perp^2}{T} J_n (J_n + J_n'') \right] \\
& - n \frac{mg}{k_x T} \Big[(J_n + J_n'') \left(\frac{m v_\perp^2}{T} J_n - \xi J_n' \right) \\
& - n^2 J_n \left(\frac{J_n}{\xi} \right)' \Big] + \frac{\varkappa_1^2}{k_x^2} \Big[n^2 \frac{m v_\perp^2}{T} \Big[J_n \\
& + \xi \left(\frac{J_n}{\xi} \right)'' \Big] J_n - \xi^2 (J_n + J_n'') J_n \Big] \\
& - \frac{\varkappa}{k_x} \frac{mg}{k_x T} \Big[n^2 \Big[\xi J_n J_n' + \xi J_n' \left(\frac{J_n}{\xi} \right)'' \\
& + \frac{\xi}{2} J_n J_n''' - J_n^2 - \xi J_n \left(\frac{J_n}{\xi} \right)'' \Big] + J_n (J_n + J_n'') \xi^2 \\
& - \frac{T k_x^2}{m \omega_c^2} \xi \left[J_n' (J_n + J_n'') + \frac{n^2 J_n}{\xi} \left(\frac{J_n}{\xi} \right)' \right] \Big] \Big\}
\end{aligned}
$$

$$
\varkappa = \frac{1}{n_0} \frac{\partial n_0}{\partial y}; \quad \varkappa_1^2 = \frac{1}{n_0} \frac{\partial^2 n_0}{\partial y^2} \qquad (4.6)
$$

$$
f_\perp = \frac{1}{n_0} \int f_0 \, d\varepsilon_z; \quad \eta_n = (\omega - n\omega_c - u_z k_x)^{-1}
$$

It is obvious from Eq. 4.6 that the term of the series with $n = 0$ reverts to zero in the case of addends of the zero and first order in respect to g and $\varkappa$ but differs from zero in the case of the addends proportional to $\varkappa_y^2$ $g\varkappa$. Hence for the low-frequency waves when $T_\perp \gtrsim T_\parallel \neq 0$, the values of the order of $\varkappa_1^2$ and $g\varkappa$ enter into σ_{xx} with considerably greater weight than do the values of the zero order and of the order of g, $\varkappa$.

Further, inasmuch as in the braces, the addends of the zero order are even functions (of) n, while those of the first order are odd, then their products times:

$$
\eta_n \approx -\frac{1}{n\omega_c} \left(1 + \frac{\omega + (gk/\omega_c)}{n\omega_c} \right)
$$

enter into σ_{xx} with a weight of varying order: zero—with a weight of about ω/ω_c, while the terms of the order $\varkappa$, g enter with a weight equalling unity. Therefore, at $\omega \ll \omega_c$, σ_{xx} acquires the form:

$$
\begin{aligned}
\sigma_{xx} = -\frac{ie^2 n_0}{m\omega_c} \sum_{n \neq 0} \int f_\perp \, d\varepsilon_\perp \Big\{ & \frac{m}{T k_x^2} \left(\omega + \frac{g k_x}{\omega_c} \right) J_n^2 \\
& + \frac{\varkappa}{k_x} \left[J_n^2 - \frac{m v_\perp^2}{T} J_n (J_n + J_n'') \right] \\
& - \frac{mg}{k_x T} \left[(J_n + J_n'') \left(\frac{m v_\perp^2}{T} J_n - \xi J_n' \right) - n^2 J_n \left(\frac{J_n}{\xi} \right)' \right] \Big\} \\
& + \frac{ie^2 n_0}{m\omega + (mgk/\omega_c)} \int f_\perp \, d\varepsilon \, \Big\{ - \frac{\varkappa_1^2}{k_x^2} \xi J_0 J_1 \\
& - \frac{\varkappa}{k_x} \frac{mg}{k_x T} \left(\xi J_0 J_1 + \frac{T k_x^2}{m\omega_c^2} J_1^2 \right) \Big\}
\end{aligned}
\qquad (4.7)
$$

We also assume that the ratio of the Larmor radius to the wave length, $Tk_x^2/m\omega_c^2 \ll 1$. Then

$$\sigma_{xx} = -\frac{i c^2 n_0}{m\omega_c^2}\left(\omega - \frac{1}{2}\frac{\varkappa k_x T}{m\omega_c} + \frac{\varkappa_1^2 \frac{T}{m} + g\varkappa}{\omega + (g k_x/\omega_c)}\right) \tag{4.8}$$

From this it follows that allowance for the terms of the second order in respect to the gradients is necessary at the frequencies:

$$\omega^2 \lesssim g\varkappa + \varkappa_1^2 \frac{T}{m} \tag{4.9}$$

Such frequencies are at the same time typical for the magnetohydrodynamic instabilities [ref. 2].

5 Flows and Density in a Low-Pressure Maxwellian Plasma

5.1 In the present section, we adduce actual expressions of $\varepsilon_{\alpha\beta}$ for an important particular case, when the function f_0 is Maxwellian, the plasma pressure is small in comparison with the magnetic pressure $(8\pi p/B_0^2 \ll 1)$, while gravitational force is lacking $(g = 0)$. We will disregard entirely the heterogeneity of the magnetic field, i.e. we will consider that $\nabla B^0 = 0$. At $8\pi p/B_0^2$, this signifies that we are excluding from the deliberation those waves having a phase velocity of the order of the velocity of diamagnetic drift, while in a consideration of the more rapid waves, we disregard the exponentially small terms in $\sigma_{\alpha\beta}$, conditioned by the interaction of the very fast particles with the wave. We so select the readout system that $E^0 = 0$.

In this case, the tensor of dielectric permeability, will consist of three parts: of a homogeneous one, $\varepsilon_{\alpha\beta}{}^0$, i.e. one coinciding with $\varepsilon_{\alpha\beta}$ for a homogeneous plasma; of a part linked with the heterogeneity of the wave amplitude, $\varepsilon_{\alpha\beta}{}^\nu$; and of a part depending on the spatial derivatives of the equilibrium distribution function, $\varepsilon_{\alpha\beta}{}^a$, i.e.:

$$\varepsilon_{\alpha\beta} = \varepsilon_{\alpha\beta}{}^0 + \varepsilon_{\alpha\beta}{}^\nu + \varepsilon_{\alpha\beta}{}^a \tag{5.1}$$

The tensor $\varepsilon_{\beta\alpha}{}^0$ was computed previously by numerous authors [refs. 5–9]. It can be written in either of two ways: in the form of the sum of an infinite series, or in an integral form, so that

$$\varepsilon_{xx}{}^0 = 1 - \sum_{\alpha,n} \frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2} W_n\right) e^{-z}\frac{n^2}{z} I_n$$

$$= 1 + \sum_{\alpha} \frac{\omega_0^2}{\omega\omega_c}\int_0^\infty e^A\left(\frac{\omega}{\omega_c} + \frac{i\varphi}{2}\zeta^2\right)\sin\varphi\, d\varphi$$

$$\varepsilon_{yx}{}^0 = -\varepsilon_{xy}{}^0 = -i\sum_{\alpha,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2} W_n\right) e^{-z}\, n\, (I_n$$

$$- I_n') = i\sum_{\alpha}\frac{\omega_0^2}{\omega\omega_c}\int_0^\infty e^A\left[z(1-\cos\varphi) - 1\right]\sin\varphi\, d\varphi \tag{5.2}$$

$$\varepsilon_{yy}^0 = \varepsilon_{xx}^0 - \sum_{\alpha,n} \frac{\omega_0^2}{\omega k_z v_T} (-i\pi^{1/2} W_n) e^{-z} 2z (I_n - I_n')$$

$$= \varepsilon_{xx}^0 + i \sum_{\alpha} \frac{\omega_0^2}{\omega \omega_c} 2z \int_0^\infty d\varphi\, e^{A} (1 - \cos\varphi)$$

$$\varepsilon_{xz}^0 = \varepsilon_{zx}^0 = \sum_{\alpha,n} \frac{\omega_0^2}{\omega \omega_c} \frac{k_x}{k_z} (-i\pi^{1/2} W_n) x_n \frac{n}{z} e^{-z} I_n$$

$$= -\sum_{\alpha} \frac{\omega_0^2}{\omega \omega_c} \frac{k_x}{k_z} \zeta^2 \int_0^\infty e^{A} \varphi \sin\varphi\, d\varphi$$

$$\varepsilon_{yz}^0 = -\varepsilon_{zy}^0 = i \sum_{\alpha,n} \frac{\omega_0^2}{\omega \omega_c} \frac{k_x}{k_z} (-i\pi^{1/2} W_n) x_n e^{-z} (I_n - I_n')$$

$$= \frac{i k_x}{k_z} \sum_{\alpha} \frac{\omega_0^2}{\omega \omega_c} \frac{\zeta^2}{2} \int_0^\infty \varphi\, d\varphi (1 - \cos\varphi) e^{A}$$

$$\varepsilon_{zz}^0 = 1 + \sum_{\alpha,n} \frac{2\omega_0^2}{\omega k_z v_T} e^{-z} x_n I_n (1 + i\pi^{1/2} x_n W_n) \qquad (5.2)$$

$$= 1 + i \sum_{\alpha} \frac{\omega_0^2}{\omega \omega_c} \int_0^\infty \left(1 - \frac{\varphi^2 \zeta^2}{2}\right) e^{A} d\varphi$$

Here

$$z = \frac{k_x^2 T}{m \omega_c^2}; \quad v_T = (2T/m)^{1/2};$$

$$x_n = \frac{\omega - n\omega_c}{k_z v_T}; \quad I_n = I_n(z)$$

$$W_n = W(x_n)$$

$$W(x) = e^{-x^2} \left(1 + \frac{2i}{\pi^{1/2}} \int_0^x e^{t^2} dt\right)$$

$$\zeta^2 = v_T^2 k_z^2 / \omega_c^2$$

$$\omega_0^2 = 4\pi e^2 n_0 / m$$

$$A = z(\cos\varphi - 1) - \frac{\zeta^2}{2} \varphi^2 + \frac{i\omega}{\omega_0} \varphi$$

I_n = the Bessel function of the imaginary argument.

The summing for α is in respect to the electrons and ions, and for n is from $-\infty$ to $+\infty$. The transfer from one form of writing to another in Eq. 5.2 is conducted with the aid of the formula:

$$i \sum_{n=-\infty}^{\infty} \frac{I_n(\lambda)}{\gamma - n} = \int_0^\infty e^{\lambda\cos\varphi + i\gamma\varphi} d\varphi$$

The tensor $\varepsilon_{\alpha\beta}^{\nu}$ is expressed by $\varepsilon_{\alpha\beta}^0$ in the following manner:

$$\varepsilon_{xx}^{\nu} = -i \frac{\partial \varepsilon_{xx}^0}{\partial k_x} \cos\psi \frac{\partial}{\partial a}; \quad \varepsilon_{yy}^{\nu} = -i \frac{\partial \varepsilon_{yy}^0}{\partial k_x} \cos\psi \frac{\partial}{\partial a}; \quad (5.3)$$

(cont'd on next page)

$$\varepsilon_{zz}^{\psi} = -\mathrm{i}\,\frac{\partial \varepsilon_{zz}^{0}}{\partial k_x}\cos\psi\,\frac{\partial}{\partial a}\,;$$

$$\varepsilon_{yx}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{yx}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\left(\varepsilon_{xx}^{0} - \varepsilon_{yy}^{0}\right)\sin\psi\right]\frac{\partial}{\partial a}$$

$$\varepsilon_{xy}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{xy}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\left(\varepsilon_{xx}^{0} - \varepsilon_{yy}^{0}\right)\sin\psi\right]\frac{\partial}{\partial a}$$

$$\varepsilon_{zx}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{zx}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\,\varepsilon_{zy}^{0}\sin\psi\right]\frac{\partial}{\partial a}$$

$$\varepsilon_{xz}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{xz}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\,\varepsilon_{zy}^{0}\sin\psi\right]\frac{\partial}{\partial a}$$

$$\varepsilon_{zy}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{zy}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\,\varepsilon_{zx}^{0}\sin\psi\right]\frac{\partial}{\partial a} \qquad (5.3)$$

$$\varepsilon_{yz}^{\psi} = -\mathrm{i}\left[\frac{\partial \varepsilon_{yz}^{0}}{\partial k_x}\cos\psi + \frac{1}{k_x}\,\varepsilon_{zx}^{0}\sin\psi\right]\frac{\partial}{\partial a}$$

We represent the tensor $\varepsilon_{\alpha\beta}^{a}$ in the form of a sum:

$$\varepsilon_{\alpha\beta}^{a} = \varepsilon_{\alpha\beta}^{\perp}\sin\psi + \varepsilon_{\alpha\beta}^{\parallel}\cos\psi \qquad (5.4)$$

The components $\varepsilon_{\alpha\beta}^{\perp}$ have the form of Eq. 5.5:

$$\varepsilon_{xx}^{\perp} = \frac{1}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-\mathrm{i}\,\pi^{1/2}W_n\right) n\, e^{-z}\left[\left(\frac{n\omega_c}{\omega}-1\right)I_n - z\left(I_n - I_n''\right)\right]$$

$$\varepsilon_{xy}^{\perp} = -\frac{\mathrm{i}}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-\mathrm{i}\,\pi^{1/2}W_n\right)e^{-z}\left[\left(I_n - I_n'\right)\left(2z + n^2 - n\,z\frac{\omega_c}{\omega}\right) + \frac{n^2}{z}I_n\right]$$

$$\varepsilon_{yx}^{\perp} = \frac{\mathrm{i}}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left\{\left(-\mathrm{i}\,\pi^{1/2}W_n\right)n^2 e^{-z}\left(I_n - I_n' + \frac{I_n}{z}\right) - z\left(I_n - I_n'\right)e^{-z}\frac{1}{x_0}\left[1 + \mathrm{i}\,\pi^{1/2}W_n\left(x_n - x_0\right)\right]\right\}$$

$$\varepsilon_{yy}^{\perp} = \frac{1}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-\mathrm{i}\,\pi^{1/2}W_n\right)e^{-z}\left\{n\left(I_n - I_n'\right) + 2\left[z\left(I_n - I_n'\right) + \frac{n^2}{z}I_n\right]\left(n - \frac{\omega_c}{\omega}z\right)\right\}$$

$$\varepsilon_{zx}^{\perp} = -\frac{1}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1 + \mathrm{i}\,\pi^{1/2}x_n W_n\right)e^{-z}I_n\, n\left(\frac{n}{z} - \frac{\omega_c}{\omega}\right)$$

$$\varepsilon_{xz}^{\perp} = -\frac{1}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1 + \mathrm{i}\,\pi^{1/2}x_n W_n\right)e^{-z}\left(I_n - I_n' + \frac{n^2}{z}I_n - \frac{n\,\omega_c}{\omega}I_n\right)$$

$$\varepsilon_{yz}^{\perp} = -\frac{\mathrm{i}}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1 + \mathrm{i}\,\pi^{1/2}x_n W_n\right)e^{-z}\left[\frac{n I_n}{z} + \left(I_n - I_n'\right)\left(n - \frac{\omega_c}{\omega}z\right)\right].$$

$$\varepsilon_{zy}^{\perp} = -\frac{\mathrm{i}}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1 + \mathrm{i}\,\pi^{1/2}x_n W_n\right)e^{-z}\left(I_n - I_n'\right)\left(\frac{\omega_c}{\omega}z - n\right)$$

$$\varepsilon_{zz}^{\perp} = \frac{1}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\,2x_n\left(1 + \mathrm{i}\,\pi^{1/2}x_n W_n\right)e^{-z}I_n\left(\frac{\omega_c}{\omega}z - n\right) \qquad (5.5)$$

Analogously for $\varepsilon_{\alpha\beta}^{\parallel}$ we have the form of Eq. 5.6:

$$\varepsilon_{xx}^{\parallel} = \frac{\mathrm{i}}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-\mathrm{i}\,\pi^{1/2}W_n\right)n^2 e^{-z}\left(I_n + I_n' + \frac{I_n}{z}\right) \qquad (5.6)$$

(cont. on next p.)

$$\varepsilon_{xy}^{1}=-\frac{i}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_x v_T}\left(-i\pi^{1/2}W_n\right)n\,e^{-z}\left[(I_n-I_n')(2z-1)+\frac{n^2}{z}I_n\right]$$

$$\varepsilon_{yx}^{1}=-\frac{1}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2}W_n\right)n\,e^{-z}z\,(I_n'-I_n'')$$

$$\varepsilon_{yy}^{1}=-\frac{i}{k_x}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2}W_n\right)e^{-z}z\left[z\,(I_n'''-3I_n''+3I_n'-I_n)+2I_n''-3I_n'+I_n\right]$$

$$\varepsilon_{xz}^{1}=\frac{i}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1+i\pi^{1/2}x_n W_n\right)n\,e^{-z}\left(I_n-I_n'+\frac{I_n}{z}\right)$$

$$\varepsilon_{zx}^{1}=\frac{i}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1+i\pi^{1/2}x_n W_n\right)n\,e^{-z}(I_n-I_n')$$

$$\varepsilon_{yz}^{1}=-\frac{1}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1+i\pi^{1/2}x_n W_n\right)e^{-z}\left[-I_n+z\,(I_n''+I_n-2I_n')\right]$$

$$\varepsilon_{zy}^{1}=\frac{1}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\left(1+i\pi^{1/2}x_n W_n\right)e^{-z}\left[\frac{n^2}{z}I_n+2z(I_n-I_n')\right]$$

$$\varepsilon_{zz}^{1}=\frac{i}{k_z}\frac{\partial}{\partial a}\sum_{a,n}\frac{\omega_0^2}{\omega\omega_c}\frac{k_x v_T}{\omega_c}x_n\left(1+i\pi^{1/2}x_n W_n\right)e^{-z}(I_n-I_n') \qquad (5.3)$$

We will not write out the integral representation of the values for $\varepsilon_{\alpha\beta}^{1}$. The contribution to $\varepsilon_{\alpha\beta}$ from the terms with the second derivatives of the distribution function in respect to coordinate is omitted.

5.2 It is clear that the heterogeneity effects should be particularly appreciable for the low-frequency oscillations ($\omega < \omega_c$), since it is specifically in this case that there are branches of oscillations, entirely indebted (in their origin) to the presence of drift velocities [ref. 2]. Under the stipulation that $\omega < \omega_c$, the contribution to $\varepsilon_{\alpha\beta}$ from the corresponding component of the plasma will be determined by the following relationships:

$$\varepsilon_{xx}^{0}=\frac{\omega_0^2}{\omega_c^2}\frac{1}{z}\left(1-I_0e^{-z}\right)$$

$$\varepsilon_{yx}^{0}=-\varepsilon_{xy}^{0}=-i\frac{\omega_0^2}{\omega\omega_c}(I_0-I_1)e^{-z}$$

$$\varepsilon_{yy}^{0}=\varepsilon_{xx}^{0}-\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2}W_0\right)2z\,(I_0-I_1)e^{-z}$$

$$\varepsilon_{xz}^{0}=\varepsilon_{zx}^{0}=-\frac{2\omega_0^2}{\omega_c^2}\frac{k_z}{k_x}\sum_{n\neq0}\frac{I_ne^{-z}}{n^2} \qquad (5.2\,a)$$

$$\varepsilon_{yz}^{0}=-\varepsilon_{zy}^{0}=i\frac{\omega_0^2}{\omega\omega_c}\frac{k_x}{k_z}(I_0-I_1)e^{-z}\left(1+i\pi^{1/2}x_0W_0\right)$$

$$\varepsilon_{zz}^{0}=\frac{2\omega_0^2}{\omega k_z v_T}e^{-z}I_0x_0\left(1+i\pi^{1/2}x_0W_0\right)$$

$$\varepsilon_{xx}^{1}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}I_1e^{-z}$$

$$\varepsilon_{yx}^{1}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega^2}z\,e^{-z}(I_0-I_1) \qquad (5.5a)$$

$$\varepsilon_{xy}^{1}=-\frac{i}{k_x}\frac{\partial}{\partial a}\left[\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2}W_0\right)2z\,e^{-z}(I_0-I_1)\right.$$
$$\left.-\frac{\omega_0^2}{\omega^2}z\,e^{-z}(I_0-I_1)\right]$$

$$\varepsilon^{1}_{yy}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_z v_T}\frac{\omega_c}{\omega}2z^2e^{-z}(I_0-I_1)(-i\pi^{1/2}W_0)$$

$$\varepsilon^{1}_{xz}=-\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}(1+i\pi^{1/2}x_0W_0)e^{-z}(I_0-I_1)$$

$$+\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\frac{k_z}{k_z}(1-I_0e^{-z})$$

$$\varepsilon^{1}_{zx}=\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\frac{k}{k_x}\sum_{n\neq 0}e^{-z}I_n\left(1-\frac{2}{n^2}\right)$$

$$\varepsilon^{1}_{yz}=-\varepsilon^{1}_{zy}=\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0}{\omega^2}(1+i\pi^{1/2}x_0W_0)e^{-z}z(I_0-I_1)$$

$$\varepsilon^{1}_{zz}=\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_z v_T}2x_0(1+i\pi^{1/2}x_0W_0)\frac{\omega_c}{\omega}zI_0e^{-z}$$

$$\varepsilon^{2}_{xx}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega_c^2}\left[\frac{1}{z}(1-I_0e^{-z})-(I_0-I_1)e^{-z}\right]$$

$$\varepsilon^{2}_{xy}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}(1+2z)(I_0-I_1)e^{-z}$$

$$\varepsilon^{2}_{yx}=\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}ze^{-z}(I_1'-I_1) \qquad \text{(5.5a) (cont'd)}$$

$$\varepsilon^{3}_{yy}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_z v_T}(-i\pi^{1/2}W_0)e^{-z}2z[I_0$$

$$-2z(I_0-I_1)] \qquad (5.6a)$$

$$\varepsilon^{3}_{xz}\approx 0 \qquad \varepsilon^{3}_{zx}\approx 0$$

$$\varepsilon^{3}_{yz}=-\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}(1+i\pi^{1/2}x_0W_0)e^{-z}$$

$$\times[-I_0+z(I_0''+I_0-2I_1)]$$

$$\varepsilon^{3}_{zy}=\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}(1+i\pi^{1/2}x_0W_0)e^{-z}2z(I_0-I_1)$$

$$\varepsilon^{3}_{zz}=\frac{i}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\frac{k_xv_T}{\omega_c}x_0(1+i\pi^{1/2}x_0W_0)e^{-z}(I_0-I_1)$$

In addition to the assumption regarding the triviality of the frequency, we also used the condition $k_z v_T \ll \omega_c$, and in this regard, we disregarded the exponentially small terms proportionally to $\exp[-(n\omega_c/k_z v_T)^2]$, corresponding to the interaction of very fast particles with the wave. Therefore (i.e. on the basis of $k_z v_T \ll \omega_c$), the relationships in Eqs. 5.2a, 5.5a, and 5.6 a in the case of large arguments of the Bessel functions, $z \gtrsim 1$, have meaning only at an adequately wide angle between the wave vector and the constant magnetic field, $k_z/k_x \ll 1$. In the opposite case, the Bessel function in these equations should be expanded into a series with respect to the small argument, $z \ll 1$. The latter, in conjunction with the stipulation that $\omega \ll \omega_c$, at the same time corresponds to the drift approximation [ref. 1], improved by a consideration of the finite Larmor radius of the particles [ref. 2]. In this regard, the tensor

of dielectric permeability of the appropriate component of the plasma has the form:

$$
\begin{aligned}
&\varepsilon_{xx}^{0}=\frac{\omega_0^2}{\omega_c^2}\\
&\varepsilon_{yy}^{0}=\frac{\omega_0^2}{\omega_c^2}-2z\frac{\omega_0^2}{\omega k_z v_T}\left(-i\pi^{1/2}W_0\right)\\
&\varepsilon_{yx}^{0}=-\varepsilon_{xy}^{0}=-i\frac{\omega_0^2}{\omega\omega_c}\left(1-\frac{3}{2}z\right)\\
&\varepsilon_{xz}^{0}=\varepsilon_{zx}^{0}=-\frac{2\omega_0^2}{\omega_c^2}\frac{k_z}{k_x}z\\
&\varepsilon_{yz}^{0}=-\varepsilon_{zy}^{0}=\frac{i\omega_0^2}{\omega\omega_0}\frac{k_x}{k_z}\left(1+i\pi^{1/2}x_0W_0\right)\left(1-\frac{3}{2}z\right)\\
&\varepsilon_{zz}^{0}=\frac{2\omega_0^2}{k_z^2v_T^2}\left(1+i\pi^{1/2}x_0W_0\right)
\end{aligned}
\tag{5.2b}
$$

$$
\begin{aligned}
&\varepsilon_{xx}^{\perp}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\frac{z}{2}\\
&\varepsilon_{yx}^{\perp}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega^2}z\\
&\varepsilon_{xy}^{\perp}=-\frac{i}{k_x}\frac{\partial}{\partial a}\left\{\frac{\omega_0^2}{\omega^2}z\left[2\frac{\omega}{k_zv_T}\left(-i\pi^{1/2}W_0\right)-1\right]\right\}\\
&\varepsilon_{yy}^{\perp}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_zv_T}\frac{\omega_c}{\omega}2z^2\left(-i\pi^{1/2}W_0\right)\\
&\varepsilon_{zx}^{\perp}=-\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\frac{k_z}{k_x}\frac{z}{2}\\
&\varepsilon_{xz}^{\perp}=-\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_c}\left(1+i\pi^{1/2}x_0W_0\right)\\
&\qquad\qquad+\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_0}\frac{k_z}{k_x}\\
&\varepsilon_{yz}^{\perp}=-\varepsilon_{zy}^{\perp}=\frac{i}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega^2}z\left(1+i\pi^{1/2}x_0W_0\right)\\
&\varepsilon_{zz}^{\perp}=\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_zv_T}2x_0\left(1+i\pi^{1/2}x_0W_0\right)\frac{\omega_0}{\omega}z
\end{aligned}
\tag{5.5b}
$$

$$
\begin{aligned}
&\varepsilon_{xx}^{\parallel}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega_c^2}\frac{7}{4}z\\
&\varepsilon_{xy}^{\parallel}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{2\omega_0^2}{\omega\omega_0}\\
&\varepsilon_{yx}^{\parallel}=\frac{1}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_0}\frac{z}{2}\\
&\varepsilon_{yy}^{\parallel}=-\frac{i}{k_x}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega k_zv_T}2z\left(-i\pi^{1/2}W_0\right)\\
&\varepsilon_{xz}^{\parallel}\approx 0\quad \varepsilon_{zx}^{\parallel}\approx 0\\
&\varepsilon_{yz}^{\parallel}=\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_0}\left(1+i\pi^{1/2}x_0W_0\right)\\
&\varepsilon_{zy}^{\parallel}=\frac{1}{k_z}\frac{\partial}{\partial a}\frac{\omega_0^2}{\omega\omega_0}2z\left(1+i\pi^{1/2}x_0W_0\right)\\
&\varepsilon_{zz}^{\parallel}=\frac{i}{k_x}\frac{\partial}{\partial a}\frac{2\omega_0^2}{\omega k_zv_T}zx_0\left(1+i\pi^{1/2}x_0W_0\right)
\end{aligned}
\tag{5.6b}
$$

We not[illegible] that the contribution from $\varepsilon_{\alpha\beta}$ [illegible] ∇ is small in comparison with the remaining part (of) $\varepsilon_{\alpha\beta}$ - as [illegible] $\partial/\partial x$); however, this contribution has the nature of an imaginary addition to the real values. Therefore, generally speaking, it can not be omitted.

5.3. In the study of the longitudinal fluctuations, it is convenient not to use the components of the tensor $\varepsilon_{\alpha\beta}$, but the component of the vector X (see eq. 3.19) in (for) the direction of the wave vector. Then the Poisson equation will acquire the form:

$$1=\frac{4\pi i}{k^2}\mathbf{X}\cdot\mathbf{k} \qquad (5.7)$$

For low pressure plasma with a Maxwell distribution in respect to velocities, this equation can be reduced to the following form: ($k \perp \nabla f_0$):

$$\begin{aligned}1 &+ \sum_{i,e}\frac{4\pi e^2 n_0}{k^2}\sum_{n=-\infty}^{\infty}\Big\{I_n e^{-z}\Big[\frac{n\omega_c}{T}\frac{i\pi^{1/2}W_n}{k_z v_T} \\ &+ \frac{1}{T}(1+i\pi^{1/2}x_n W_n)\Big] \\ &+ \frac{1}{n_0}\frac{\partial}{\partial y}I_n e^{-z}\frac{k_x n_0}{m\omega_c}\Big(1-\frac{n\omega}{z\omega_c}\Big)\frac{i\pi^{1/2}W_n}{k_z v_T}\Big\}=0\end{aligned} \qquad (5.8)$$

If temperature does not depend on y, then

$$\begin{aligned}1 &+ \sum_{i,e}\frac{4\pi e^2 n_0}{k^2}\sum_{n=-\infty}^{\infty}I_n e^{-z}\Big\{\Big[\frac{n\omega_c}{T} \\ &+ \frac{\varkappa k_x}{m\omega_c}\Big(1-\frac{n\omega}{z\omega_c}\Big)\Big]\frac{i\pi^{1/2}W_n}{k_z v_T} \\ &+ \frac{1}{T}(1+i\pi^{1/2}x_n W_n)\Big\}=0 \\ \varkappa &= \frac{1}{n_0}\frac{\partial n_0}{\partial y}\end{aligned} \qquad (5.8\,a)$$

If the frequency of oscillations is low ($\omega \ll \omega_{ci}$), in place of eq. 5.8a we will then have:

$$\begin{aligned}1+\sum_{i,e}\frac{4\pi e^2 n_0}{k^2 T}\Big[1+i\pi^{1/2}x_0 W_0 I_0 e^{-z}\Big(1-\frac{k_x v_{dp}}{\omega}\Big)\Big]=0 \\ v_{dp}=-\frac{\varkappa T}{m\omega_c}\end{aligned} \qquad (5.9)$$

Just as eq. 5.8a, this equation was also developed in the paper by Kadomtsev & Timofeyev [Ref. 10] by use of another method.

6 Fluctuations of the Alfvén and Ion-Sonic Types in a Low-Pressure Heterogeneous Plasma

In the present section, we have derived a dispersion equation describing the slow waves, of which the phase velocity is much less than the Alfven velocity. The frequency of oscillations is assumed low in comparison with the cyclotron frequency of the ions. The plasma pressure will be considered slight as compared with the pressure of the magnetic field, $\beta = 8\pi p/B^{o2} \ll 1$.

We shall examine the waves propagating almost transverse to the constant

magnetic field ($k_z \ll k_y$) and transverse to the direction of the heterogeneity ($\psi = \pi/2$). In a homogeneous plasma, we are aware of two types of low-frequency slow waves with $k_z \ll k_y$: the Alfven and the ion-sonic ones, and also one fast wave, namely the magnetic-sonic one. In a heterogeneous plasma, there are modified most intensively just specifically the slow waves, especially if their phase velocity is comparable with or less than the velocity of Larmor drift. We derive the dispersion equation for such waves, using the Maxwell equation written as follows:

$$\frac{c^2}{\omega^2}(\nabla \times \nabla \times \mathrm{E})_\alpha = \varepsilon_{\alpha\beta}{}^{\mathrm{d}} E_\beta \qquad (6.1)$$

The symbol $^{\mathrm{d}}$ connotes the presence in $\varepsilon_{\alpha\beta}$ of the differential operators $\partial/\partial a$.

Assuming $k \perp \nabla f_o$ ($\psi = \pi/2$), we get from eq. 6.1:

$$\begin{aligned}
&N^2\left(\cos^2\theta - \frac{1}{k^2}\frac{\partial^2}{\partial y^2}\right)E_x - N^2\cos\theta\sin\theta\, E_z \\
&\qquad + iN^2\frac{1}{k}\frac{\partial E_\gamma}{\partial y} = \varepsilon_{x\beta}{}^{\mathrm{d}} E_\beta \\
&N^2 E_\gamma + iN^2\frac{1}{k}\left(\sin\theta\frac{\partial E_x}{\partial y} + \cos\theta\frac{\partial E_z}{\partial y}\right) = \varepsilon_{\gamma\beta}{}^{\mathrm{d}} E_\beta \\
&-N^2 E_x\cos\theta\sin\theta + N^2\left(\sin^2\theta - \frac{1}{k^2}\frac{\partial^2}{\partial y^2}\right)E_z \\
&\qquad + \frac{i\cos\theta}{k}N^2\frac{\partial E_\gamma}{\partial y} = \varepsilon_{z\beta}{}^{\mathrm{d}} E_\beta \qquad (6.2) \\
&N^2 = c^2 k^2/\omega^2, \qquad \cos\theta = k_z/k
\end{aligned}$$

We shall consider that $\cos^2\theta \ll 1$, $\sin^2\theta \approx 1$. Since $\varepsilon_{\gamma\gamma} \propto c^2/v_A^2$, while $v_\phi \equiv \omega/k \ll v_A$, $(v_A^2 = B^{02}/4\pi n_0 m_i)$, $N^2 \gg \varepsilon_{\gamma\gamma}$.

From the second equation of system 6.2, it then follows:

$$E_\gamma = \frac{1}{N^2}\varepsilon_{\gamma\beta}{}^{\mathrm{d}} E_\beta - \frac{i}{k}\left(\frac{\partial E_x}{\partial y} + \cos\theta\frac{\partial E_z}{\partial y}\right), \quad \beta = 1, 3 \qquad (6.3)$$

Substituting this expression for E_γ into the other two equations of 6.2, we get:

$$\begin{aligned}
&\left(N^2\cos^2\theta - \varepsilon_{xx} - \frac{\varepsilon_{\gamma x}{}^{\mathrm{d}}\varepsilon_{x\gamma}{}^{\mathrm{d}}}{N^2} + \frac{i}{k}\frac{\partial}{\partial y}\varepsilon_{\gamma x}{}^{\mathrm{d}} + \frac{i}{k}\varepsilon_{x\gamma}{}^{\mathrm{d}}\frac{\partial}{\partial y}\right)E_x \qquad (6.4) \\
&-\left(N^2\cos\theta + \varepsilon_{xz}{}^{\mathrm{d}} + \frac{\varepsilon_{x\gamma}{}^{\mathrm{d}}\varepsilon_{\gamma z}{}^{\mathrm{d}}}{N^2} - \frac{i}{k}\frac{\partial}{\partial y}\varepsilon_{\gamma z}\right. \\
&\qquad\left. - \frac{i\cos\theta}{k}\varepsilon_{x\gamma}{}^{\mathrm{d}}\frac{\partial}{\partial y}\right)E_z = 0 \\
&-\left(N^2\cos\theta + \varepsilon_{zx}{}^{\mathrm{d}} + \frac{\varepsilon_{z\gamma}{}^{\mathrm{d}}\varepsilon_{\gamma x}{}^{\mathrm{d}}}{N^2} - \frac{i\cos\theta}{k}\frac{\partial}{\partial y}\varepsilon_{\gamma x}{}^{\mathrm{d}}\right. \\
&\qquad\left. - \frac{i}{k}\varepsilon_{z\gamma}{}^{\mathrm{d}}\frac{\partial}{\partial y}\right)E_x + \left(N^2 - \varepsilon_{zz} - \frac{\varepsilon_{z\gamma}{}^{\mathrm{d}}\varepsilon_{\gamma z}{}^{\mathrm{d}}}{N^2}\right. \\
&\qquad\left. + \frac{i\cos\theta}{k}\frac{\partial}{\partial y}\varepsilon_{\gamma z}{}^{\mathrm{d}} + \frac{i\cos\theta}{k}\varepsilon_{z\gamma}{}^{\mathrm{d}}\frac{\partial}{\partial y}\right)E_z = 0
\end{aligned}$$

We consider the frequency of oscillations to be low as compared with the cyclotron frequency of the ions ($\omega \ll \omega_{ci}$). From the results in Sect. 5, it then follows:

$$
\begin{aligned}
&\frac{i}{k}\frac{\partial}{\partial y}\varepsilon_{yx}{}^{d} + \frac{i}{k}\varepsilon_{xy}{}^{d}\frac{\partial}{\partial y} \approx \frac{i}{k}\frac{\partial \varepsilon_{yx}}{\partial y}\\
&\frac{i}{k}\frac{\partial}{\partial y}\varepsilon_{yz}{}^{d} + \frac{i}{k}\varepsilon_{zy}{}^{d}\frac{\partial}{\partial y} \approx \frac{i}{k}\frac{\partial \varepsilon_{yz}}{\partial y}\\
&\varepsilon_{xz}{}^{d} - \frac{i}{k}\frac{\partial}{\partial y}\varepsilon_{yz}{}^{d} \approx 0\\
&\varepsilon_{zx}{}^{d} - \frac{i}{k}\varepsilon_{zy}{}^{d}\frac{\partial}{\partial y} \approx -\frac{2i}{k}\varepsilon_{zy}\frac{\partial}{\partial y}
\end{aligned}
\tag{6.5}
$$

With the aid of eq. 6.5, we can be assured that the field E_z in the second equation in 6.4 is not differentiated in respect to y and it can be expressed simply by E_x:

$$
\begin{aligned}
E_z = &\left(N^2 - \varepsilon_{zz} - \frac{\varepsilon_{zy}\varepsilon_{yz}}{N^2} + \frac{i\cos\theta}{k}\frac{\partial}{\partial y}\varepsilon_{yz}\right)^{-1}\\
&\times\left(N^2\cos^2\theta + \frac{\varepsilon_{zy}\varepsilon_{yx}{}^{d}}{N^2} - \frac{i\cos\theta}{k}\frac{\partial}{\partial y}\varepsilon_{yx}{}^{d}\right.\\
&\left. - \frac{2i}{k}\varepsilon_{zy}{}^{d}\frac{\partial}{\partial y}E_x\right)
\end{aligned}
\tag{6.6}
$$

Having excluded from the first equation in 6.4 the field E_z, we derive a differential equation for E_x; abandoning a study of the spatial dependence of the field E_{ox}, we arrive (see analogous formulation of the problem in ref. 1) at the "dispersion equation":

$$
\begin{aligned}
&N^2\cos^2\theta - \varepsilon_{xx} - \frac{\varepsilon_{yx}{}^{d}\varepsilon_{xy}}{N^2} + \frac{i}{k}\frac{\partial\varepsilon_{yx}}{\partial y}\\
&-\left[N^2\cos\theta + \frac{\varepsilon_{xy}{}^{d}\varepsilon_{yz}}{N^2} - \frac{i\cos\theta}{k}\varepsilon_{xy}{}^{d}\frac{\partial}{\partial y}\right]\\
&\times\left(N^2 - \varepsilon_{zz} - \frac{\varepsilon_{zy}\varepsilon_{yz}}{N^2} + \frac{i\cos\theta}{k}\frac{\partial\varepsilon_{yz}}{\partial y}\right)^{-1}\\
&\times\left(N^2\cos\theta + \frac{\varepsilon_{zy}\varepsilon_{yz}}{N^2} - \frac{i\cos\theta}{k}\frac{\partial\varepsilon_{yz}}{\partial y}\right) = 0
\end{aligned}
\tag{6.7}
$$

Having estimated the $\varepsilon_{\alpha\beta}$ -value and discarding the terms of the order β and $(kv_{pr}/\omega_i)\beta$, we can arrive at a more compact equation:

$$
\left(\varepsilon_{xx} - \frac{i}{k}\frac{\partial\varepsilon_{yx}}{\partial y} - N^2\cos^2\theta\right)\varepsilon_{zz} - N^2\left(\varepsilon_{xx} - \frac{i}{k}\frac{\partial\varepsilon_{yx}}{\partial y}\right) = 0
\tag{6.8}
$$

Using the expressions for $\varepsilon_{\alpha\beta}$ from Sect. 5, we will write this equation as follows:

$$
\begin{aligned}
&(\omega^2 - \omega\,\omega_i{}^{*} - k_z{}^2\langle v_A{}^2\rangle)\left[1 + \sum_{i,e}\left(\varepsilon_{xx}{}^{0}\right.\right.\\
&\left.\left. + \frac{1}{k_x}\frac{\partial}{\partial y}\frac{\omega_c}{\omega}\,2\,\varepsilon_{xz}{}^{0}\right)\right] - c^2k^2\left(1 - \frac{\omega_i{}^{*}}{\omega}\right) = 0
\end{aligned}
\tag{6.9}
$$

$$\langle v_A^2\rangle = \frac{z_i}{1 - I_0(z_i)e^{-z_i}} v_A^2 \qquad (6.9)\ \text{(cont'd)}$$

$$v_A^2 = \frac{H^2}{4\pi n_0 m_i}$$

$$\omega_i^* = -\frac{k_x T_i \varkappa_i}{m_i \omega_{ci}} \qquad \varkappa_i = \frac{\partial}{\partial y} \ln \frac{z_i}{\langle v_A^2\rangle}$$

$$\varepsilon_{zz}^0 = \frac{2\omega_0^2}{k_z^2 v_T^2} I_0 e^{-z}\left[1 + i\pi^{1/2}\frac{\omega}{k_z v_T} W\left(\frac{\omega}{k_z v_T}\right)\right]$$

It is handy to study eq. 6.9 in two marginal cases, when the argument W = either a small function ($\omega \ll k_z v_T$) or large ($\omega \gg k_z v_T$) and when (as is known) this function has a simple analytical form. It can be demonstrated that eq. 6.9 does not have solutions with $\omega \ll k_z v_{Ti}$, if

$$\cos\theta \gtrsim \beta_i \varkappa \varrho_i \qquad (6.10)$$

Therefore considering that eq. 6.10 is fulfilled, we will assu[me] that $\omega \gg k_z v_{Ti}$ and we will represent ε_{zz}^0 for ions in the form:

$$\varepsilon_{zz}^0 = -\frac{\omega_0^2}{\omega^2}\left(1 + \frac{3Tk_z^2}{m\omega^2}\right) I_0 e^{-z}$$
$$\omega_0^2 = 4\pi e^2 n_0/m \qquad (6.11)$$

We will not be concerned with the minor effects associated with exp $(-\omega^2/k_z^2 v_{Ti}^2)$, hence we have dropped from the right hand part of eq. 6.11 the exponentially small imaginary terms.

The ratio $\omega/k_z v_{Te}$ (the argument of the electron W -- function) can be either low or high. In the first case, we can assume W to equal unity, so that in this connection, the ε_{zz}^0-value for electrons will equal:

$$\varepsilon_{zz}^0 = \frac{2\omega_0^2}{k_z^2 v_T^2} I_0 e^{-z}\left(1 + i\pi^{1/2}\frac{\omega}{k_z v_T}\right) \qquad (6.12)$$

while at $\omega \gg k_z v_{Te}$, for the electron ε_{zz}^0-value, we will have the expression 6.11.

Using Eqs. 6.11 and 6.12, we find that at $\omega \ll k_z v_{Te}$, eq. 6.9 will yield:

$$\begin{aligned}(\omega^2 - \omega\omega_i^* - k_z^2\langle v_A^2\rangle)\Big\{\omega^2 + \omega\frac{k_x T_e}{m_e\omega_{ce}}\frac{\partial}{\partial y}\ln n_0 \\ - \frac{k_z^2 T_e}{m_i} I_0(z_i)e^{-z_i}\Big[1 \\ + \frac{k_x T_i}{m_i\omega_{ci}\omega}\frac{\partial}{\partial y}\ln[I_0(z_i)e^{-z_i} n_0 T_i]\Big] \\ + \frac{i\pi^{1/2}\omega^3}{k_z v_{Te}}\left(1 + \frac{k_x T_e}{m_e\omega_{ce}\omega}\frac{\partial}{\partial y}\ln n_0 T_e^{-1/2}\right)\Big\} \\ = z_i\frac{T_e}{T_i}k_z^2 v_A^2\,\omega(\omega - \omega_i^*)\end{aligned} \qquad (6.13)$$

while at $\omega \gg k_z v_{Te}$ /175

$$(\omega^2 - \omega\omega_i^* - k_z^2 v_A^2)\left(\omega + \frac{k_x T_e}{m_e \omega_{ce}} \frac{\partial}{\partial y} \ln n_0 T_e\right) = -\frac{c^2 k^2}{\omega_{oe}^2} \omega^2 (\omega - \omega_i^*) \quad (6.14)$$

Equations 6.13 and 6.14 are used as points of departure in the report conducted by the author in cooperation with L.I. Rudakov /ref. 11/.

On the basis of these equations, in the work indicated there was developed an overall pattern, describing the low-frequency ($\omega \ll \omega_{ci}$) slow ($v_\phi \equiv \omega / k \ll v_A$) waves in a low pressure heterogeneous plasma. It was demonstrated that, just as in a homogeneous plasma, the number of branches of fluctuations, $\omega = \omega(k)$, equals four, two of which can be classified as ion-sonic waves, while the other two can be classified as Alfvén waves.

The waves being described in eq. 6.8, in case of the real wave vector, have a complex frequency, the imaginary part of which can be both negative (as in a homogeneous plasma) as well as positive, which corresponds to the buildup of oscillations. It is possible that the buildup of such waves can represent a hazard in the experiments in the confinement of plasma.

7 Conclusion

The method offered by us permits the investigation of all types of small-scale ($k \gg \varkappa$) oscillations of a heterogeneous plasma located in a heterogeneous magnetic field with straight lines of force. Some of the results of the present paper have already been used for the investigation of certain types of low-pressure plasma oscillations in reports 4, 11, & 12. It is proposed that certain problems be considered subsequently.

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